Honors Analysis
Chapter 3

Don't even think about getting 30 minutes behind!

Exam C. ..er 2

• • • • • • • • • • • • • • •

1 R/C Pg 109-113

• • • • • • • • • • • • • • •

2 Write out Q & A to Oral Ex pg 113 1 – 11

3/ 7

Important Lecture
3 unLIMITed Enjoyment section I

Lecture # 1

3/ 10

4, #5 unLIMITed Enjoyment section II, III

****Tomorrow ****
Lecture # 2
Def of Continuity &
1st Def of Der

3/ 11

Important Lecture

6 Pg 119 1 - 10, also graph pblms numbered 5 - 9 Pg 125 13 - 24. Use abscissa = 2 (That means the x value is two.)

3/ 12

7 Pg 125 Replace text instructions with: Find $f'(x)$ using 2nd def on problems 1 - 12. **Also** for each of 1 - 12, find the x–value where $f'(x) = 0$

Lecture # 3 2nd def

3/13

8 Pg 130 1 –18 (1,7,13; 2,8,14; etc.) Also # 9 Page 136 1, 2, 4, 6

Lecture # 4
Fast derivatives and pg 136 pblms 3, 5, 7

3/ 14

10,11 F, F ', F "
Finish the handout

Lecture:
Do part I of F, F ', F "

3/ 17

#12,13 Number Lines-R-Us

End points and cusps are at least local extrema

3/18

Begin
#14, 15, 16, 17, 18
"Just a Few Interesting Problems."

3/19

Minimum Day
Enjoying

"Just a few ..."

3/20

Minimum Day
Turn in "Just a Few ...

• • • • • • • • • • • • • • •

19 "Find the Pretender"

• • • • • • • • • • • • • • •

Begin **"Mr. Summary"**
20, 21, 22, 23, 24

3/ 21

Turn in #19

3/ 24

Industrial Strength
el Quizzo
value: 4 assignments!

3 / 25

3/26

Put last year's exam on the chalk boards

3/27

EXAM CHAPTER III

3/ 28

$$(x^3 \pm y^3) =$$
$$(x \pm y)(x^2 \mp xy + y^2)$$

- - - - - - - - -

$$(x \pm y)^3 =$$
$$(x^3 \pm 3x^2y \mp 3xy^2 \pm y^3)$$

2003

You didn't do this last year!
This chapter is the backbone of the course. You *must* make every effort to understand *completely*.
Master this stuff!

unLIMITed Enjoyment

Honors Analysis

Mr. Mumaugh

I Find the following limits:

a) $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$ b) $\lim_{h\to 0}\frac{h^2}{6-\sqrt{36-h^2}}$ c) $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}$ d) $\lim_{x\to 1}\frac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{2}}}{x+2}$

e) $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$ f) $\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}$ g) $\lim_{h\to 0}\frac{(a+h)^2-a^2}{h}$ h) $\lim_{h\to 0}\frac{\sqrt{a+h}-\sqrt{a}}{h}$

i) $\lim_{x\to -2}(x+3)^{99}$ j) Find the limit **as h goes to zero** for problems 38, 39, 40 on page 11 in the text.

k) Clear parentheses: $(a^2-ab+b^2)(a+b)$ l) Clear parentheses: $(p-q)(p^2+pq+q^2)$

m) Show long division: $2x^2+5x-3\overline{)6x^3+19x^2+x-6}$ n) $\lim_{x\to -3}\frac{6x^3+19x^2+x-6}{2x^2+5x-3}$

o) $\lim_{x\to -\frac{2}{3}}\frac{6x^3+19x^2+x-6}{2x^2+5x-3}$ p) $\lim_{x\to \frac{1}{2}}\frac{2x^2+5x-3}{6x^3+19x^2+x-6}$

II Find the following limits.

a) $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$ b) $\lim_{x\to 0}\frac{1-\sqrt{2x^2+x+1}}{x}$ c) $\lim_{x\to 2}\frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ d) $\lim_{x\to 1}\frac{x^2-2x-3}{x^2-1}$

e) $\lim_{x\to 1}\frac{x^2+2x-3}{x^2-1}$ f) $\lim_{h\to 0}\sqrt{1+\frac{1}{h}}-\sqrt{\frac{1}{h}}$ g) $\lim_{x\to -2}\frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ h) $\lim_{x\to 2}\frac{x^4-16}{x-2}$

i) $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$ j) $\lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{x}$ k) $\lim_{x\to -2}\frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x-2}$ l) $\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

m) $\lim_{x\to 1}\left(\frac{1}{1-x^3}+\frac{1}{3(x-1)}\right)$ n) $\lim_{h\to 0}\left(\frac{(x+h)^3-x^3}{h}\right)$

III Find the limits:

a) $\lim_{x\to 2^-}\sqrt{2-x}$ b) $\lim_{x\to 0^+}\sqrt{x}$ c) $\lim_{x\to 0^-}\sqrt{x}$ d) $\lim_{x\downarrow 1}\frac{x-1}{|x-1|}$

e) $\lim_{x\uparrow 1}\frac{x-1}{|x-1|}$ f) $\lim_{x\to 1}\frac{x-1}{|x-1|}$ g) $\lim_{x\to 4^+}\sqrt{x-4}-2$ h) $\lim_{x\to 0}\frac{\sqrt{x^2+9}-3}{x^2}$

i) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$ j) $\lim_{x\to 3}\sqrt{x-5}$ k) $\lim_{x\to 0}\left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

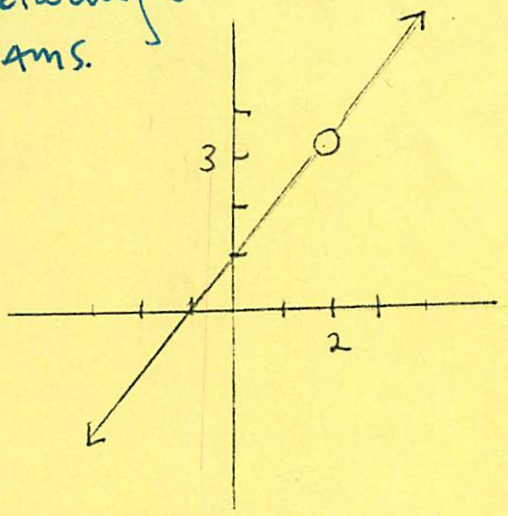
l) Re-write as a piece meal function: $F(x)=\frac{x+|x|}{|x|}$ m) Graph $F(x)=\frac{x+|x|}{|x|}$ n) $\lim_{x\to 0}\frac{x+|x|}{|x|}$

n) Explain (in English) ***exactly*** (no examples, please) what is meant by: $\lim_{x\to a} f(x)=L$. Be sure to include how "left-hand limits" , "right-hand limits", and "limits" are related and how they are different

1ST Lecture
After returning ch 2 Exams.

Given : $F(X) = \frac{(X + 1)(X - 2)}{(X - 2)}$

Let's assume $X < 2$.
As X starts from -1,000 and gets bigger, F(X) gets bigger. Consider the picture at the right. You can see that F(X) will steadly increase without change with only ONE EXCEPTION. That exception is at (2,3).



So, as X gets bigger toward 2, from below (That is, from smaller than 2) F(X) nears 3.

Let's assume $X > 2$.
We can also say as X gets smaller toward 2, from above (This is from larger than 2) F(X) nears 3.

Because of both of the above statements, we could say "The limit as X goes to 2 from below and the limit as X goes to 2 from above is 3. Therefore, The limit of F(X) as X goes to 2 is 3."

This is written briefly as : $\lim_{X \to 2} F(X) = 3$

This was easy to see because of the graph but we shouldn't have to draw a graph for every function in order to determine its limit.

Consider another function : $G(X) = X + 1$.

G is exactly like F in all but one place. I know this because I simply did something to F(X) which didn't really change its value --- much --- I reduced it. G is a different function because it has a different Domain and a different Range. Then, G is very much like F with that one exception.

If I know what G does as X nears some value then I say that F probably does the same thing as G.

Since G has no Domain restriction at X = 2,
I can say : $\lim_{X \to 2} G(X) = 3$

Thus, as $X \to 2$, $G \to 2 + 1 = 3$. Sure, I just found G(2).

So, what is a limit? Well, its a bit hard to define. A limit is a number. It is a "Range" value which may or may not be achievable by the function in question. As in our example, F(X) did not have 3 in its range, but 3 was the limit of the function. G(X) also had a limit of 3 and 3 was in its range.

A limit is "taken" as the domain of the function nears some specific number. The value the function is nearing at that time is the Limit.

OK. That was painless. Now we "know" what a limit is, how do we find a limit? ---- I thought you'd never ask. See page 2.

like the Nike slogan)

HOW TO FIND LIMITS

Remember that a limit is the value the function nears as X nears some particular spot.

Usually nobody will ask you to find a limit of a function as it nears some "well defined" domain value because all you have to do is to find F(that domain value) and you have the limit.

Ah ha, --- I think he is leading up to something ---- Thus - "if there is NO Reason NOT to put the value IN, Just Do it!"

Generally, we want to know the limit of a function as X approaches some domain restriction.

Your job will be to determine what the function does as X nears that restriction.

You need to "change" the given function into another function which "acts" like the original and then take the limit."

Ok, How do you change the given function? ----- carefully, of course.

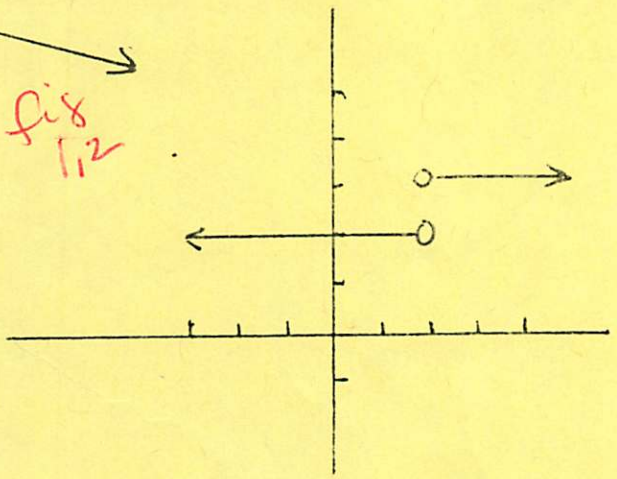
The two most common methods are:

1. "reducing the orginal function" like I did in my example.
2. "multiplying the original function by one and then reducing." something which is equal to 1

If neither of these methods work (for us) then we conclude that the limit does not exist.

Let's go back to our original graph. Notice that I said "as X goes to 2 from below" and "as X goes to 2 from above". Both limits are required to make the general state ment that "the limit exists".

Let's consider a counter example for a moment. This is a piece meal function (which everone is so expert at now -- it's AFTER the exam)



fix 1,2

What is the limit as $X \to 2^-$? (that means "from below") 2

What is the limit as $X \to 2^+$? 3

Since $\lim_{x \to 2^-} F(x) = 2$ AND

$\lim_{X \to 2^+} F(x) = 3$

we say $\lim_{X \to 2} F(x)$ Does Not Exist.

If we were asked simply to find the "left hand limit" I would say $\lim_{X \to 2^-} F(x) = 2$

NOTICE : I was not concerned with the fact that the function was **NOT** defined at the point in question.

Ah, ---- yes. Question?

Did everybody hear that question?

Ok. It was "What do you mean about multiplying by one then reducing?"

Let's see if I can think up a good example

Given $F(x) = \dfrac{\sqrt{x+3} - 2}{x - 1}$ $\dfrac{\sqrt{\ }+2}{\sqrt{\ }+2}$ $\dfrac{x+3-4}{(x-1)(\sqrt{\ }+2)}$ $\lim\limits_{x\to 1} \dfrac{1}{\sqrt{x+3}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

Find $\lim\limits_{x\to 1} F(x)$

Multiply top AND bottom by $\sqrt{x+3} + 2$.

This is a fracton, of value 1 which may cause some pleasant changes in the original function.

Try it and see what you get.

Do you get the limit to be $\frac{1}{4}$? If not look at someone else's paper to see how they got it.

The general rule is to get another function which is identical as the original functon, except at the domain value in question. You may be able to just evaluate the function if the X value is not also a domain restriction, you may have to multiply by one on top AND on the bottom then simplify, you may have to just simplify and reduce. Each problem is unique and must be approached carefully.

What was that? Oh yes, Thanks for reminding me.

Home work is section I on page 4. Tomorrow is Section II on page 4. The next day is page 5. (lets see, page 5 is Wednesday), Thursday is page 6 and page 7 on Friday. Monday is new material and possibly another thrilling "silent lecture".

Just so you know my heart is in the right place.

ENJOY

END of lecture #1

"CONTINUOUS ENJOYMENT"

Begin lecture #2

"3-Step Process"

4th day of unit.

DEFINITION: $\lim_{x \to a} F(x) = F(a)$ ② ① ③

Translation to English: The point must be defined (F(a)) and the function must approach that Range value from both sides of the Domain value.

Dick & Jane Translation: If you can draw the graph of F(x) without lifting the pencil, the function is continuous.

Generally we a really deciding if a function is continuous at a point with coordinates (a, F(a)).

Example #1 $F(x) = x + 1$. Is it continuous at $x = 3$?

$\lim_{x \to 3^+} F(x) = 4$ $\lim_{x \to 3^-} F(x) = 4$ SO $\lim_{x \to 3} F(x) = 4$

AND $F(x) = 4$ ..... so .... YES, F(x) continuous at 4.

If F(x) had not been 4, Not continuous.
If Either the left or right hand limit had not existed or were not equal then the Limit, itself, would not exist. Thus F(x) could not equal the limit so F would NOT be continuous.

Example #2

$$F(x) = \begin{cases} 4 & \text{if } x \neq 5 \\ 3 & \text{if } x = 5 \end{cases}$$

Is F continuous? No. Limit will be 4 but F(x) = 3 so not continuous at x = 5.

Example #3

$$F(x) = \begin{cases} \dfrac{(x-1)(x+2)}{x-1} & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$$

Is F continuous? Yes. Limit is 3 (using the top line because the limit never "really" gets to 1) and F(1) is also equal to 3.

Of course the real problem is that the PROOF of continuity depends on the ability to use Limits. So, Limits are still important and are worth the extra effort to get the concept into permanent memory!!

Another, and VERY IMPORTANT use of limits follows.

Heavy duty stuff on the next page ---- not for weak hearts!

(All of my students are so adventurous that even Harrison Ford is jealous !)

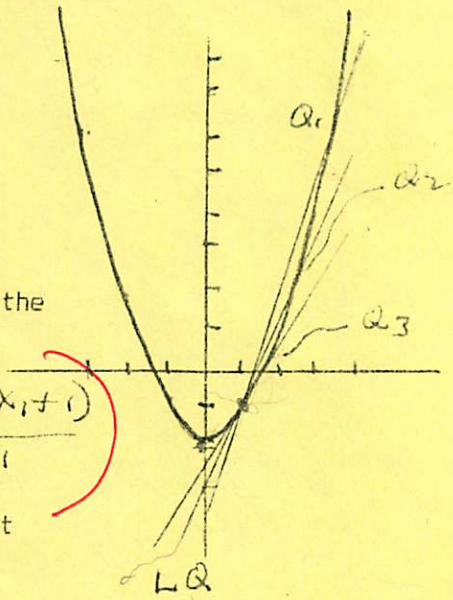
ie.

TURN TO THE NEXT PAGE

We are concerned with the **SLOPE OF THE LINE** TANGENT to F(x) at some point **P**.

As we will find out, there are some very interesting properties of this particular tangent line. Rather than ruin the surprise to come, I will only say first we learn WHAT "IT" IS then we will discuss what "IT" can do.

Consider the function $F(x) = x^2 - 2$, the point $P(1, -1)$ and the point Q_1 at $(x_1, x_1^2 - 2)$.



See the diagram at the right =======>

I have drawn the chord, L_Q

You can compute the slope of L_Q using the standard slope formula and the two points given at the beginning of the problem.

I get $x_1 + 1 = m = \dfrac{x_1^2 - 2 + 1}{x_1 - 1} = \dfrac{x_1^2 - 1}{x_1 - 1} = \dfrac{(x_1 - 1)(x_1 + 1)}{x_1 - 1}$

If I re-choose Q_2 to be closer to **P** along the graph of F(x), I would get $x_2 + 1$.

Again I re-choose Q_3 then Q_4 and so on until Q_n

Q_n will almost be the same point as **P**.

This process of choosing Q_i is what "taking the limit" is all about.

Each time I was able to get a slope.

The last time (n^{th} time) I have: $\lim_{x \to 1} \dfrac{F(x) - F(1)}{x - 1}$

This becomes in reality $\lim_{x \to 1} \dfrac{(x^2 - 2) - (1^2 - 2)}{x - 1} =$

$\lim_{x \to 1} \dfrac{x^2 - 2 + 1}{x - 1} = \lim_{x \to 1} \dfrac{x^2 - 1}{x - 1} = \lim_{x \to 1} x + 1 = 2$

Thus 2 is the slope of the tangent line at the point $P(1, -1)$.

This is written $F'(1) = 2$. You say "F prime of 1 equals 2"

$F'(1)$ is called the **DERIVATIVE OF F** at $x = 1$.

$\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a} = f'(a)$

Since we know the slope is 2, and the point is (1, -1), we can determine the equation of the line.

$- Y_1 = M (X - X_1)$
$Y + 1 = 2 (X - 1)$
$Y + 1 = 2X - 2$
$Y = 2X - 3.$

NO I don't want any of those Jepson jobs

You know, -1 + b = m(X - R), or whatever.
Just use a simple formula which ALWAYS works!

We define a "differentiable function" as one that is continuous at the point P and F(x) is a straight line or at least F(x) is a "gently" sloping curve (as contrasted by a "sharp corner" like in an absolute value function).

THE (FIRST) DEFINITION OF THE DERIVATIVE at a point (a, f(a))

$$\lim_{x \to a} \frac{F(x) - F(a)}{x - a}$$

(a, f(a))

Where "a" is the point in question. After you evaluate the limit, you will have the slope of the tangent line to F(x) at the point (a, f(a)).

Let's...see..... there was something else.....

Oh, well ....
..... I'll think of it on page 4.

I know I have not told you too much. I just want you to try to apply the definitions as I have given them to you.

Homework: page 119 1 - 9 Also graph pblms 5 - 9.
page 125 1 - 24 (1,13; 2,14; 3,15; etc)

Until next time ------ the silent lecture is over for now.

Now show
Given $f(x) = |x-2|$
I Prove f is cont
II Prove f'(2) DNE
next page.

This lecture is NOT over — go to next page for the finish.

Give next page lecture.

"CONTINUOUS ENJOYMENT"

Begin Lecture #3 but include in Lecture #2

Given $F(x) = |x + 2|$

1. Prove F(x) is continuous at $x = -2$

2. Prove F(x) does NOT have a derivative at $x = -2$

1. The definition of continuity says: $\lim_{x \to a} F(x) = F(a)$

As applied to this problem we have $\lim_{x \to -2} |x + 2| = |-2 + 2|$

We must check the approach from "below -2" and from "above -2" because the function changes "in its nature" at $x = -2$.

First, for $x < -2$

We know that $(x + 2)$ will be negative so we can remove the absolute value signs ---- as long as we replace the "inside" with "- inside".

$$\lim_{x \uparrow -2} -(x + 2) = \lim_{x \uparrow -2} -x - 2 = -(-2) - 2 = +2 - 2 = 0$$

Second, for $x > -2$

We know that $(x + 2)$ will be positive so we can remove the absolute value signs ---- by erasing them.

$$\lim_{x \downarrow -2} (x + 2) = \lim_{x \downarrow -2} x + 2 = (-2) + 2 = -2 + 2 = 0$$

So since $\lim_{x \to -2} F(x) = 0$ and $F(-2)$ also $= 0$, we have **PROVEN** that F(x) is continuous at $x = -2$

2. Prove $F'(-2)$ does not exist.

Because I want the derivative AT A SPECIFIC POINT (ie -2) I will use the "first" definition of the derivative:

$$\lim_{x \to a} \frac{F(x) - F(a)}{x - a}$$

As applied to this problem: $\lim_{x \to -2} \frac{|x + 2| - |-2 + 2|}{x - (-2)}$

First I want to simplify the above limit: $\lim_{x \to -2} \frac{|x + 2|}{x + 2}$

finish ph/lm that ends 2nd Lecture.

Re: Problem IV on
f f' f''

Letting an idea of collection of f f' f'' III, IV and Handout of "Numberlikes..."
R W

There are at least two ways to find f'(x)
Yours probably looks like this

(1) $\lim_{h\to 0} \dfrac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h}$

Pontant → $\lim_{h\to 0} \left[\frac{1}{h}\right]$ [fill in]

$\lim_{h\to 0} \frac{1}{h}$ [common denom]

"and it gets worse from here, Doesn't it?"

"consider: $\frac{x^2+1}{x} = x+\frac{1}{x}$

to $\lim_{h\to 0} \frac{1}{h}\left[x+h+\frac{1}{x+h} - \left(x+\frac{1}{x}\right)\right]$

"Does this look easier?"

finish to get $\frac{x^2-1}{x^2}$

"Do we learn anything? $1-\frac{1}{x^2}$

$f'' = \lim_{h\to 0} \frac{1}{h}\left[1 - \frac{1}{(x+h)^2} \text{ etc}\right]$

to get $\frac{2x}{x^4} = \frac{2}{x^3}$

(2) Because we have a Domain Restriction, we must show approach to it from both directions

$\lim_{x\to 0^-} \frac{x^2+1}{x} \to -\infty$

"as x goes toward zero from below f(x) becomes very Negative"

(3) "what are critical values of x?
only -1, and 1. f'(0) ≠ 0
so zero is not a critical value.

Show # lims ← 1ST Time students see # lims used in this MANNER!

f' + 0 + ∄ − 0 +
-2 -1 0 1 2

f'' − 0 +
-2 -1 0 1 2

(4) f' says + 0 − at 1st critical value what does that mean?
(-1, -2) max f(x).

(5) Notice f''(-1) ≠ 0 ∴ (-1, -2) max f'' test.

(6) Noted $(-2, -\frac{5}{2})$ min ENDPT INC from it

7) $f(x) = x^3 - x^2 - x \quad x(x^2 - x - 1)$

$f'(x) = 3x^2 - 2x - 1$

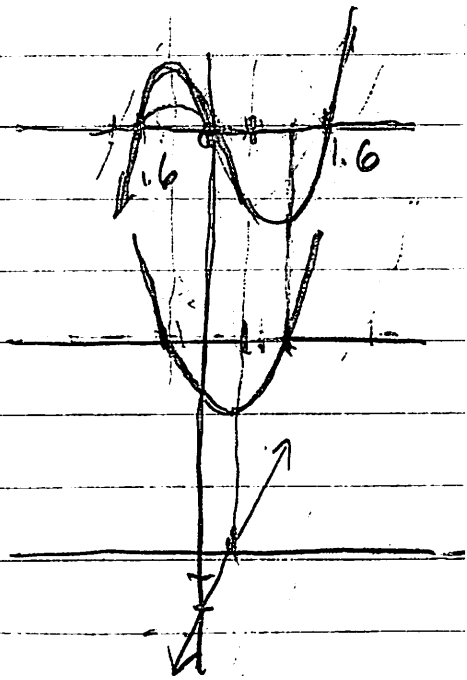
$(3x+1)(x-1)$

$f''(x) = 6x - 2$

$2(3x-1)$

$\frac{1 \pm \sqrt{1+4}}{2}$

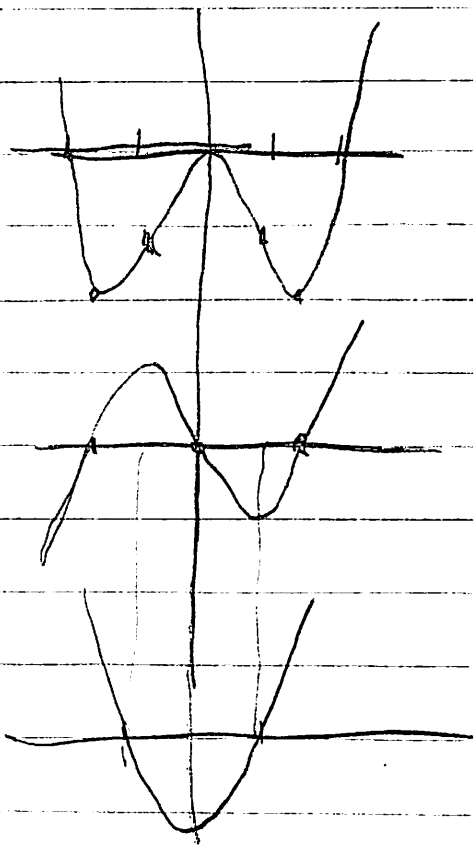
$\frac{1 \pm \sqrt{5}}{2}$



8) $f(x) = x^4 - 4x^2 = x^2(x^2 - 4)$

$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$

$f''(x) = 12x^2 - 8 \quad 4(3x^2 - 2)$



f : + 0 − @ − 0 +

$-2 \quad 0 \quad 2$

f' : − 0 + 0 − 0 +

$-\sqrt{2} \quad 0 \quad \sqrt{2}$

f'' : + 0 − 0 +

$-\sqrt{\frac{2}{3}} \quad 0 \quad \sqrt{\frac{2}{3}}$

Pg 136

④ $f(x) = x^3 + 3x^2 - 1$

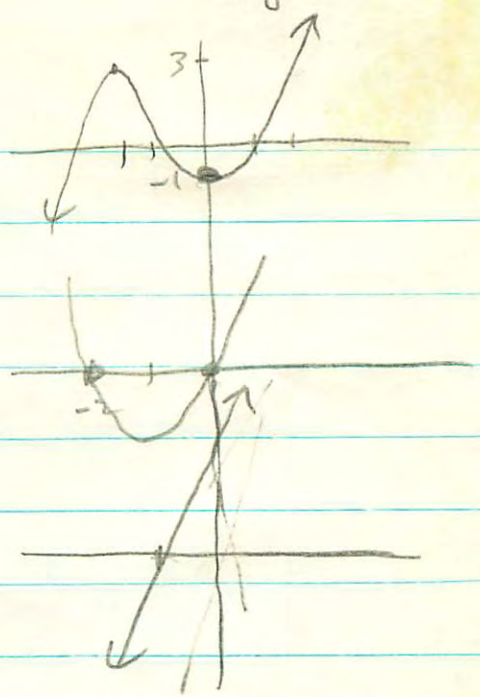
$f'(x) = 3x^2 + 6x \quad 3x(x+2)$

$f''(x) = 6x + 6$

$f(-2) = -8 + 12 - 1$

f' $+ \; 0 \; - \; 0 \; +$

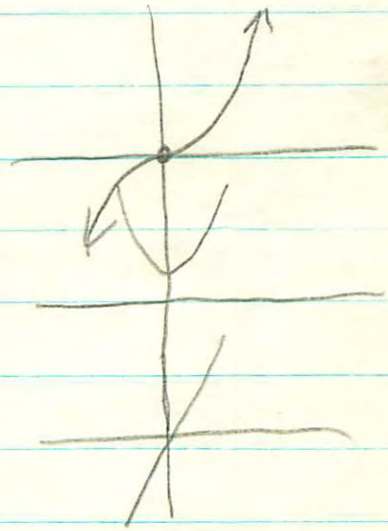
$-2 \quad 0$



⑤ $f(x) = x^3 + x \quad x(x^2+1)$

$f'(x) = 3x^2 + 1 \quad > 0 \; \forall x$

$f''(x) = 6x$



⑥ $f(x) = x^3 + 1$

$f'(x) = 3x^2$

$f''(x) = 6x$

Shifted & changed.

(7) $f(x) = x^3 - x^2 - x + 1$

$x^2(x-1) - (x-1)$

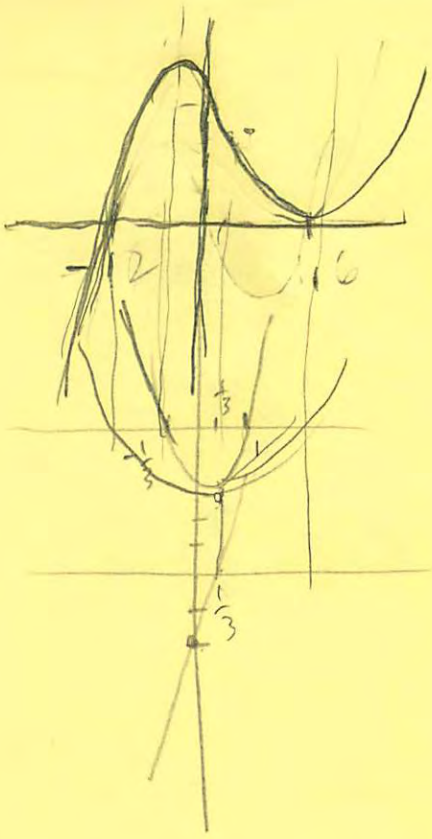
$(x^2-1)(x-1)$

$(x+1)(x-1)^2$

$f'(x) = 3x^2 - 2x - 1$

$(3x+1)(x-1)$

$f''(x) = 6x - 2$



$x = \frac{1 \pm \sqrt{1+4}}{2}$

$\frac{1 \pm \sqrt{5}}{2}$

-1.2

$+1.6$

$f(x) = (x+1)(x-1)^2$ $\quad x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$ $\quad (3x+1)(x-1)$

$f''(x) = 6x - 2$

Lecture

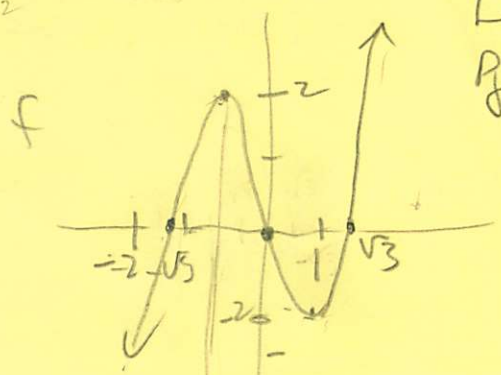
③ $f(x) = x^3 - 3x$

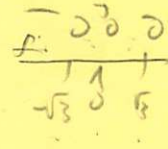
$x(x^2 - 3)$

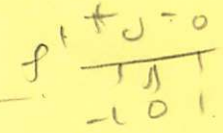
$f'(x) = 3x^2 - 3$

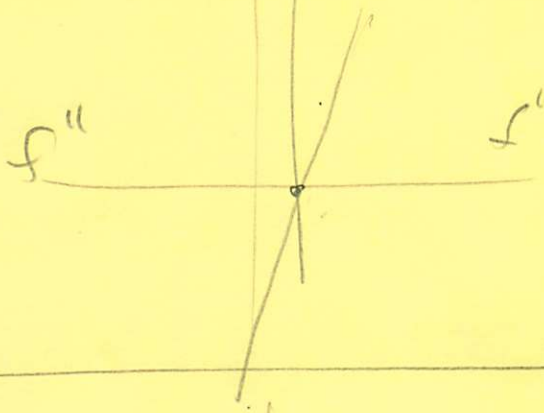
$3(x-1)(x+1)$

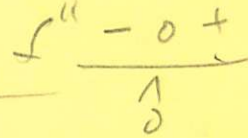
$f''(x) = 6x$











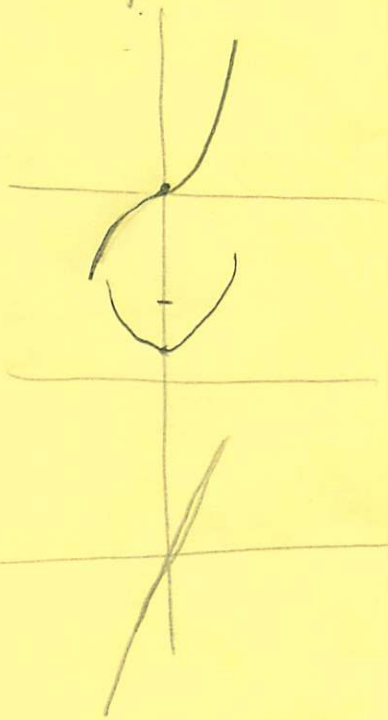
Draw all 3 problems. Start with f'' then f' and then f. Then "what is common" for all 3 problems."

⑤ $f(x) = x^3 + x$

$x(x^2+1)$

$f'(x) = 3x^2 + 1$

$f''(x) = 6x$



① when $f' > 0$ f is increasing
$f' < 0$ f is decreasing
$f'' > 0$ f is concave up
$f' > 0$ f' is increasing
$f'' < 0$ f is concave down
$f' < 0$ f' is decreasing
$f' = 0$ extreme value in f
$f'' = 0$ extreme value in f'

CONTINUOUS ENJOYMENT

Given : $F(x) = x^3 + 3x^2 - 9x - 2$

A) Find $F'(x)$
B) Find $F(-2)$ and $F'(-2)$
C) Find the equation of the tangent line to F at $x = -2$
D) Find the coordinates, if any, where the tangent line is horizontal.

"Is that all? too easy to bother with ..... huh? ..... OK ..... we'll do it anyway .....

A $F'(x) = 3x^2 + 6x - 9$

B

F		1	3	-9	-2	
-2	1	1	-11	20	=	$F(-2)$
-2	1	-1	-9		=	$F'(-2)$

C $Y - Y_1 = m(X - X_1)$ ← insist on this line.

$Y - 20 = (-9)(X - (-2))$
$Y - 20 = -9(X + 2)$
$Y - 20 = -9x - 18$
$Y = -9x + 2$

D set $3x^2 + 6x - 9 = 0$ (insist on "set")

$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x = -3 \quad x = 1$

F		1	3	-9	-2	
1	1	4	-5	-7	=	$F(1)$
-3	1	0	-9	25	=	$F(-3)$

The tangent line is horizontal at (1,-7) and at (-3,25).

OK ---------- turn to page 129 ORAL EXERCISES

1. 2X 2. 6X 3. 0 4. 1 5. 2 6. $10X^4$ 7. $-2x$ 8. $2x + 2$ 9. $4 - 2x$ 10. $3x^2 - 4x$

11. $3x^2 + 4x + 3$ 12. $100x^{99}$

13. The derivative of a linear function (ie x to first power) is the constant function.

14. The derivative of a quadradic function (ie x to the second) is a linear function.

HOMEWORK----------actually HOMEFUN--------

PAGE 130 1 - 18 1,7,13; 2,8,14; ETC Do them like the above example.
ALSO 22, 23

continued on next page

Let's try G(x)

$G(x) = x^3 + 3x^2 + 3x^1 + 1x^0$

$G'(x) = 3*1\ x^{3-1} + 3*2\ x^{2-1} + 3*1\ x^{1-1} + 1*0\ x^{0-1}$

$G'(x) = 3x^2 + 6x + 3$

$H'(x) = 4x^3 + 12x^2 + 12x + 4$

QUICK METHOD OF FINDING THE DERIVATIVE

The derivative of x^n is : $n\ x^{n-1}$

This is usually written : $\frac{d(x^n)}{dx} = nx^{n-1}$

You may use the above method of finding the derivative when:

1. You have a polynomial.

AND

2. If I did NOT say "using the definition".

As long as I'm on the topic of time savings, let's look at another area

You do remember synthetic substitution ? (Yes !!!)

Good. Like everything else in math, there is more than one useage for each "tool".

Given : $F(x) = x^4 - 2x^3 - 3x^2 + 7x + 3$

$F'(x) = 4x^3 - 6x^2 - 6x + 7$

$F(2) = 2^4 - 2(2)^3 - 3(2)^2 + 7(2) + 3 = 16 - 16 - 12 + 14 + 3 = 5$

$F'(2) = 4(2)^3 - 6(2)^2 - 6(2) + 7 = 32 - 24 - 12 + 7 = 3$

real
3 Rows

BUT notice :

line 1			1	-2	-3	7	3	
line 2		2	1	0	-3	1	5	= F(2)
line 3		2	1	2	1	3		= F'(2)

I used line 1, F(x) to get line 2 ---- as usual.
I used line 2 to get line 3. Notice that I stopped "one position short of the end".

Hey, Kids, what time is it? ----------

"Its EXAMPLE TIME !!! Yeaaaaaaaaaaaaaaaaaaaaaaaa"

(on page 9, that is)

Begin Lecture #4

Just for fun, let's do some algebra II.

$F(x) = (x + 1)^2$ $\quad G(x) = (x + 1)^3$ $\quad H(x) = (x + 1)^4$

For each of the above functions, clear the parentheses.

$F(x) = x^2+2x+1$ $\quad G(x) = x^3+3x^2+3x+1$ $\quad H(x) = x^4+4x^3+6x^2+4x+1$

Using the 2nd definition of the derivative and the functions without parentheses, find F′, G′, and H′.

$F'(x) = 2x+2$ $\quad G'(x) = 3x^2+6x+3$ $\quad H'(x) = 4x^3+12x^2+12x+4$

You should notice a pattern when you compare F and F′, G and G′ etc.

1. The prime is one degree less.
2. "Multiply the exponent times the coefficient and reduce the exponent by one . . .. ."

Does statement #2 above make sense?

If it does, then you probably cleared the parentheses properly AND you probably computed the derivative properly.

Check my work against your work!

$F(x) = x^2 + 2x + 1$

$$\lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) + 1 - x^2 - 2x - 1}{h}$$

$$\lim_{h\to 0} \frac{x^2 + 2hx + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$$

$$\lim_{h\to 0} \frac{2hx+h^2+2h}{h}$$

$$\lim_{h\to 0} 2x + h + 2 = 2x + 2$$

$F(x) = x^2 + 2x + 1$

The exponent, 2, times the coefficient, 1, is 2, reduce the 2 by 1 to get 2x

the exponent, 1, times the coefficient, 2, is 2, reduce the 1 by 1 to get exponent of zero gives $2x^0$, ie 2.

the exponent, 0, (on the term 1) ie 1 x^0. Zero times 1 etc. but zero times anything is zero so that term disappears.

Still not clear????? see the next page

Pg 1 of let 3 15

As on page 5, the absolute value changes in its nature so we will use "from above" and "from below".

When $x < -2$ we get $\lim_{x \uparrow -2} \frac{-(x+2)}{x+2} = \lim_{x \uparrow -2} -1 = -1$

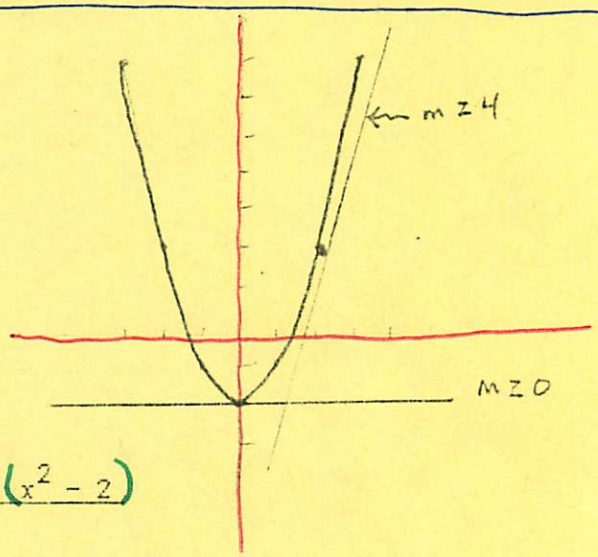
When $x > -2$ we get $\lim_{x \downarrow -2} \frac{(x+2)}{x+2} = \lim_{x \downarrow -2} 1 = 1$

Clearly, -1 does not equal $+1$ so this limit does NOT exist.

BUT "this limit" was the definition of the derivative, so we CAN say that the derivative DOES NOT EXIST.

NEW PROBLEM

Find $F'(x)$ given $F(x) = x^2 - 2$



Notice: I am asked for a function --- that is, NOT for the derivative at a specific point so I will use the second definition of the derivative.

$$\lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

First we need $F(x+h) = (x+h)^2 - 2 = x^2 + 2hx + h^2 - 2$

So the derivative becomes: $\lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 2 - (x^2 - 2)}{h}$

$$\lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} 2x + h = 2x = F'(x)$$

Consider the picture above. If you draw a tangent line at $x = 2$ then I can say that $F'(2) = 4$. So the slope of that line is 4.

If you draw the tangent line at x equal zero, $F'(0) = 0$ --- ie the line is horizontal.

Anywhere you draw a tangent line to $F(x)$, I can use $F'(x)$ to determine the slope of that tangent line.

This property, as it turns out, is of VITAL importance to us.

Well, all good things must come to an end ----- (bad things too, I guess)

Any way, this the end of another thrilling "silent lecture"

HW: Go back over yesterday's homework and on at least half of the problems, use the second definition of the derivative to find the slope.

ALSO do problems page 125,126 25-27, 30, 31

Tomorrow we get some really useful information !!

Last Time

There was something else I meant to tell you ~~yesterday~~.

There is another definition of the derivative.

Let's review a moment.

I told you that $\lim_{x \to a} \frac{F(x) - F(a)}{x - a}$ was the derivative of F at x = a.

That was correct. It is the derivative of F AT x = a.

This form of the definition of the derivative usually yields a number.

This number is the SLOPE OF THE TANGET LINE to the function F at the point (a,f(a)).

One problem with this form of the definition (and an asset at the same time) is that is gives the slope at a specific point.

Version #2 of the definition of the derivative yields a FUNCTION which is called the derivative of F, (more commonly called F′ ie F-prime).

THE SECOND DEFINITION OF THE DERIVATIVE

$$\lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

Remember, this definition will yield a FUNCTION !!!

You can evaluate this function at any value of x from the domain of F and the result will be the slope of the tangent line to F at the point (x,f(x)).

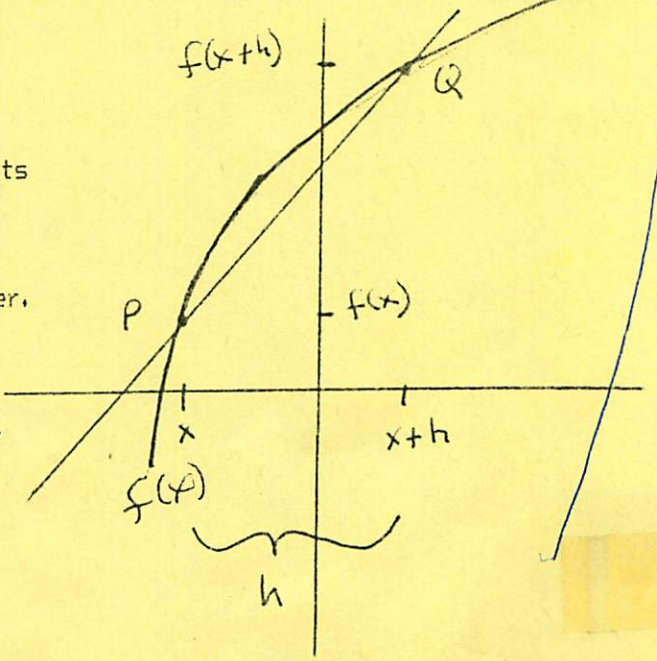
In reality there is only one definition of the derivative. I want to show you, first, that this "second" definition is really the same as the original.

Notice the point P has coordinates (x,F(x))
The first Q has coordinates ((x+h), F(x+h))
h is the horizontal distance from x to x+h.
If you use the "definition of slope", with the above points you will get the fraction to which we apply the limit in this second definition.

As Q is re-picked closer to P, h is also getting smaller.

In the 2nd definition notice that $h \to 0$.

Each version of the definition has its specific purposes. I want to show some of them on the following pages.



II

Carefully draw F ' and F " on the graphs.

Show the corresponding points of A, B. C, D, etc

Justify the locations of A', B' C' etc using complete sentences.

Determine the DEGREE of F(x).

(hint : $f'(x)$ and $f''(x)$ are continuous.)

III

Sketch a smooth curve of F (x) illustrating the following :

(Each problem uses a unique axis system and is independent of any other problem.)

a) $f(1)=3,\ f'<0\ \forall\ x<1;\ f'>0\ \forall\ x>1$

b) $f(1)=3,\ f''<0\ \forall\ x<1;\ f''>0\ \forall\ x>1$

c) $f'>0\ \forall\ x<2;\ f'<0\ \forall\ x>2;\ f(2)=3$

d) $x>0\ \ f(1)=0;\ f'(x)=\frac{1}{x}$

(hint : What is $f''(x)$ and what does it tell you?)
Use the def of the derivative to find out . . .

IV

Given: $f(x)=\frac{x^2+1}{x}$

$D_f\ \ -2\le x\le 2;\ x\neq 0$

A) Find $f'(x)$, using the second definition of the derivative.

B) Find $f''(x)$, using the second definition of the derivative.

C) Make a number line for $f(x)$.

D) Make a number line for $f'(x)$.

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F) What are the coordinates (of $f(x)$) of the critical values.

G) What are the coordinates of (of $f(x)$) $\big(-2, f(-2)\big)$ and $\big(2, f(2)\big)$?

H) List the coordinates and specifically identify all extrema.

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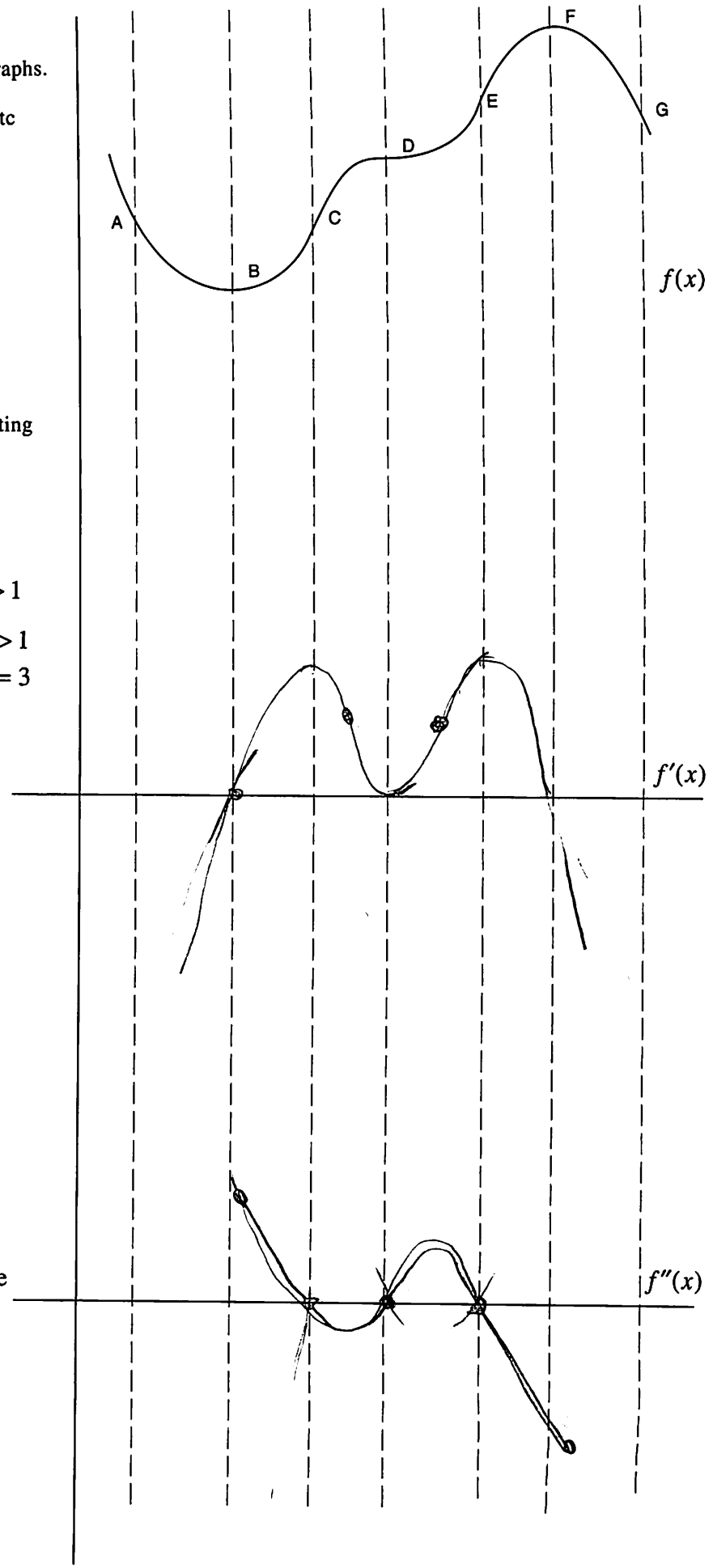
K) Sketch $f(x)$



f f' f''

Mr. Mumaugh

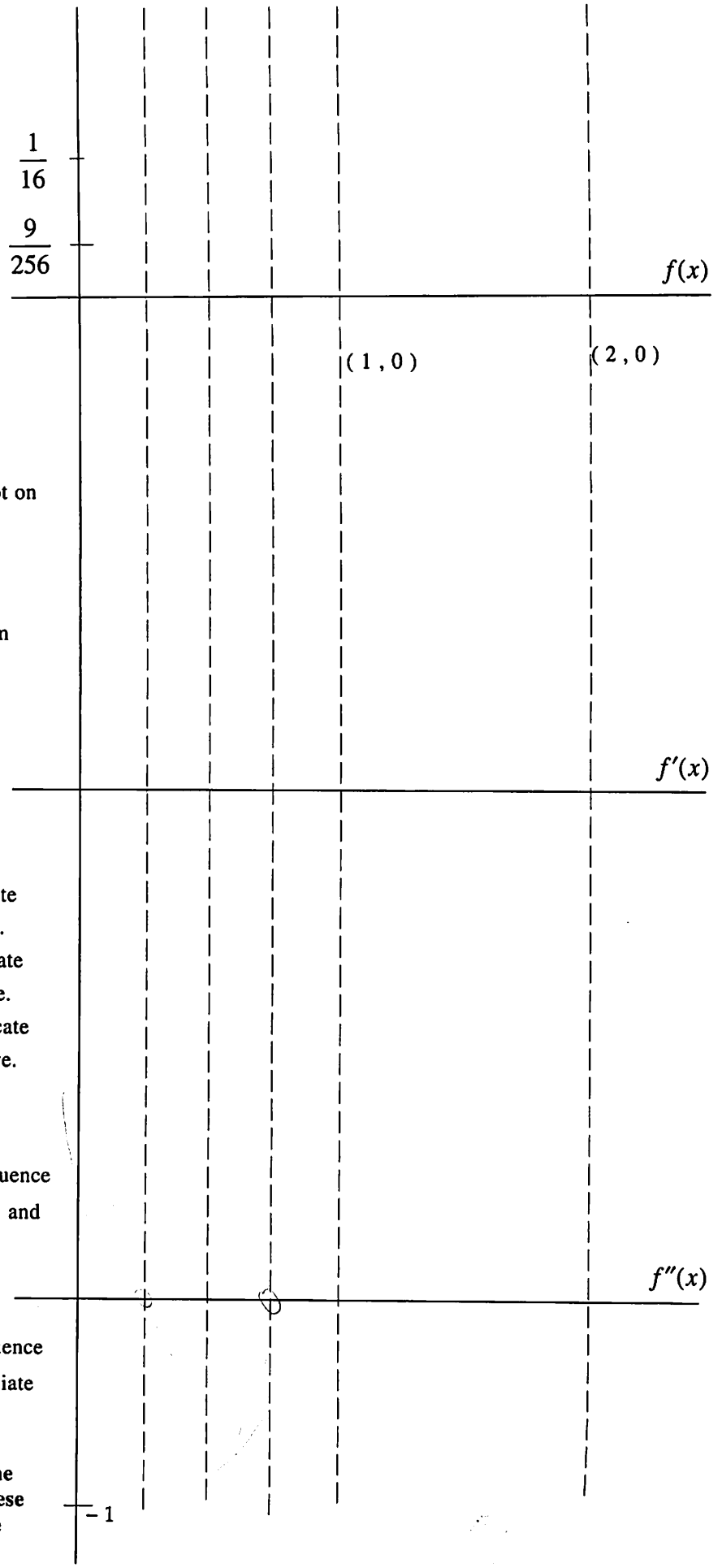
Given: $f(x) = x^4 - 2x^3 + x^2$

$f''(x) = 0 \rightarrow x \in \{\frac{1}{4}, \frac{3}{4}\}$

$f'(\frac{1}{4}) \approx \frac{1}{2}$; $f'(\frac{3}{4}) \approx -\frac{1}{2}$

$f''(x)$ has minimum at $(\frac{1}{2}, -1)$

1. Express $f(x)$ in factored form; determine ***all*** zeros of $f(x)$. Plot those points on the top axis.
2. Find $f'(x)$ and write in factored form.
3. Find all critical values of x. $f'(x) \overset{\text{set}}{=} 0$ and solve. Plot the zeros on the middle axis.
4. Write the **coordinates** of all critical values. Plot on the top axis.
5. Find $f''(x)$. Normally you would find these zeros now. However, the zeros were given above. We shall use them. Plot zeros on bottom axis.
6. Put values found in #5 into $f(x)$. Write coordinates. Plot on top axis.
7. Put values found in #5 into $f'(x)$. Write coordinates. Plot on middle axis.
8. On a number line, locate zeros of $f(x)$. Indicate where $f(x)$ is positive and where it is negative. On a number line, locate zeros of $f'(x)$. Indicate where $f'(x)$ is positive and where it is negative. On a number line, locate zeros of $f''(x)$. Indicate where $f''(x)$ is positive and where it is negative.
9. Sketch all three functions on their axes.
10. Explain how the values (+ or –) of $f''(x)$ influence $f(x)$ and $f'(x)$. i.e. What behavior in $f(x)$, and $f'(x)$ do you associate with $f''(x) > 0$? ; $f''(x) > 0$?
11. Explain how the values (+ or –) of $f'(x)$ influence $f(x)$ i.e. What behavior in $f(x)$ do you associate with $f'(x) > 0$? ; $f'(x) > 0$?
12. **The results of the above 11 problems are of the utmost importance !! Write a summary of these results AND your conclusions. Use complete sentences and be VERY complete.**



Mr. Mumaugh

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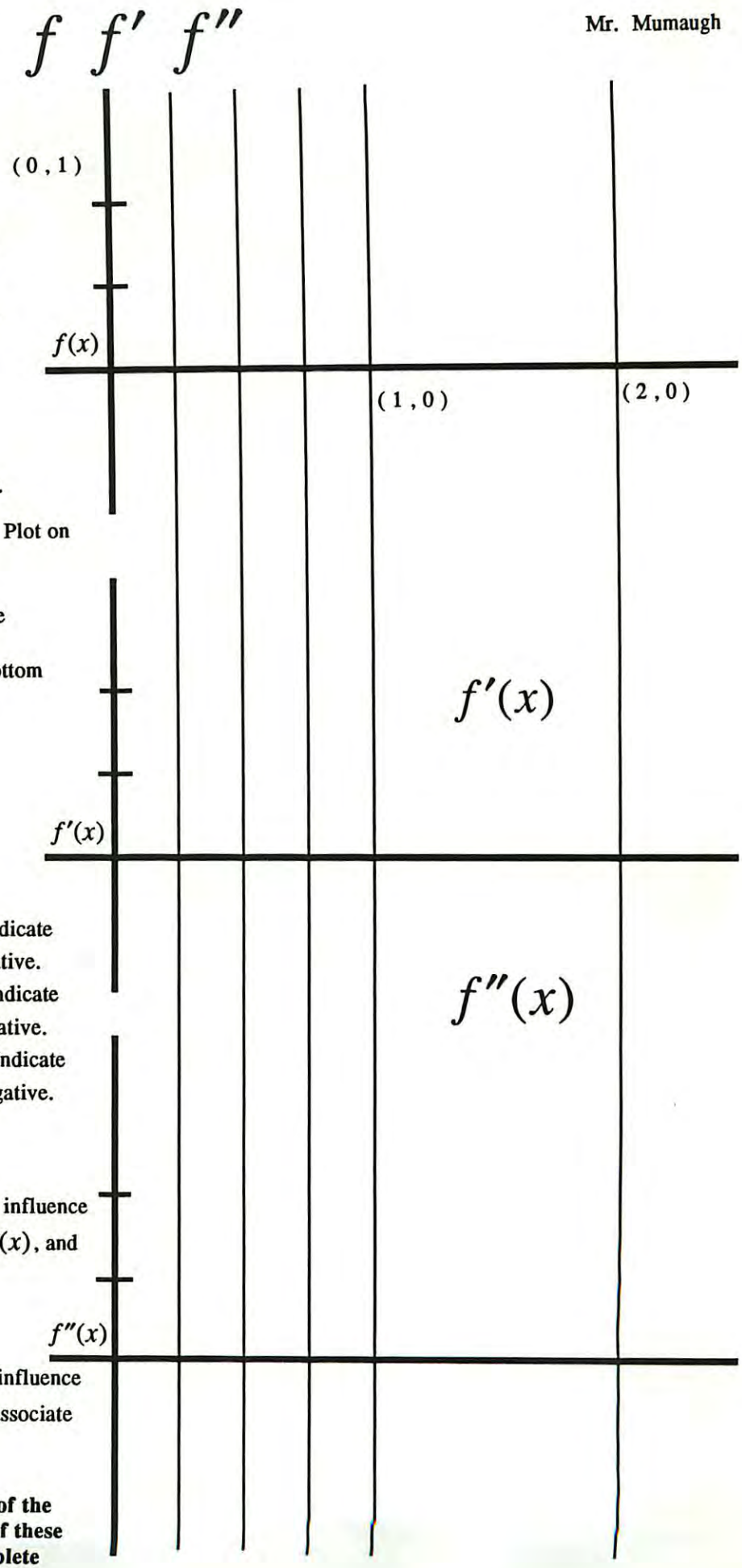

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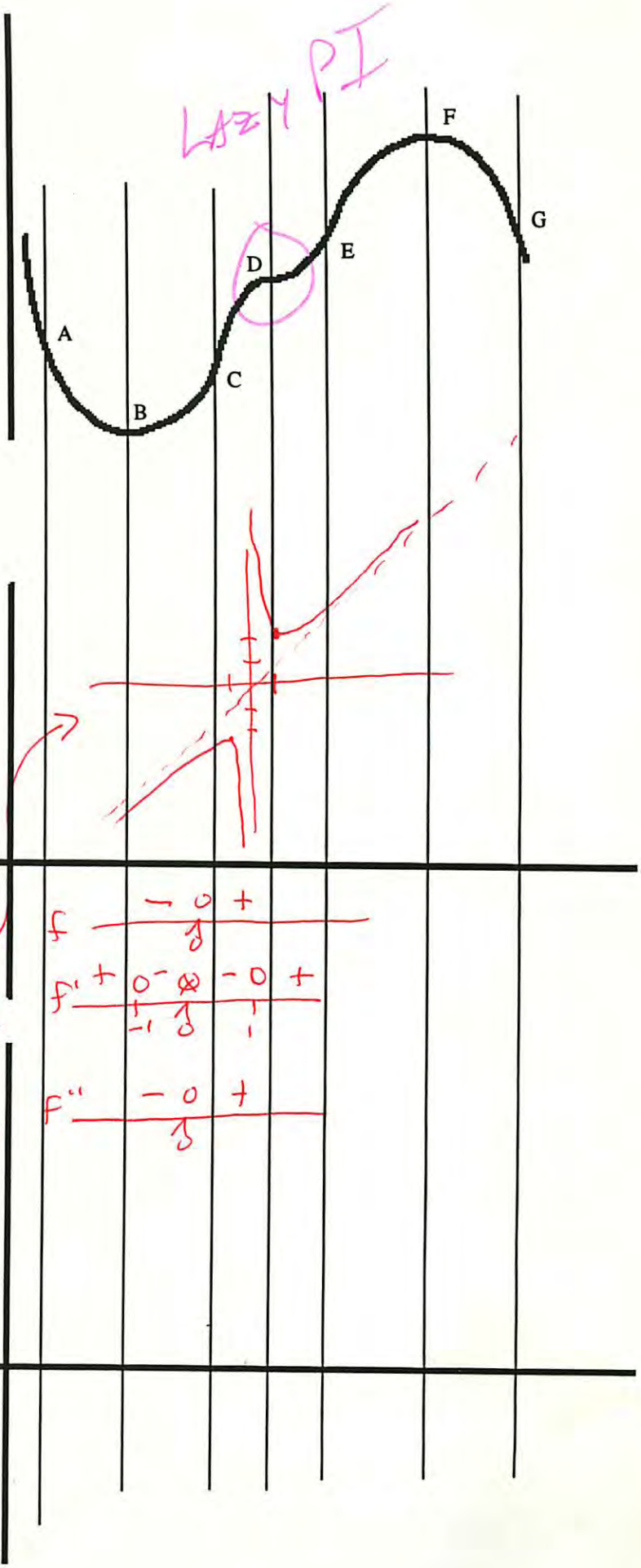

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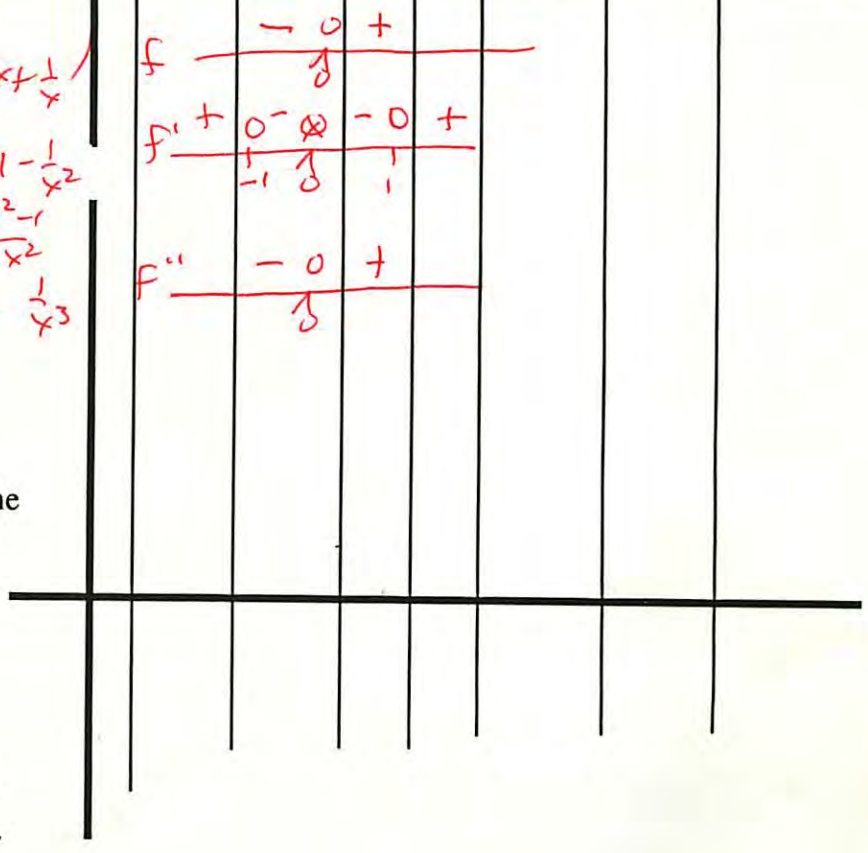


Mr. Mumaugh
Analysis Ch 3

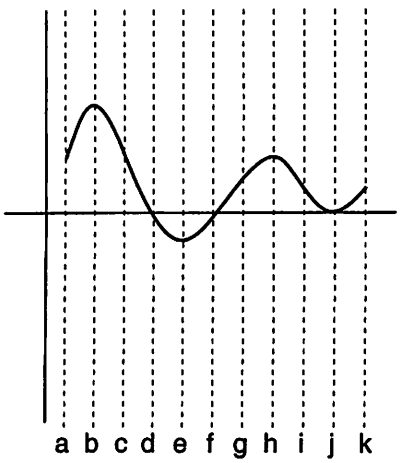
Number Lines– Я –Us

On the number line sheet provided, **mark** the f f' f'' number lines.

On separate paper, describe all max, min and PI for $f(x)$ and include proper justification.

(for example: $(e, f(e))$ max f' test or
$(a, f(a))$ min End Pt. increasing from it.)

a)



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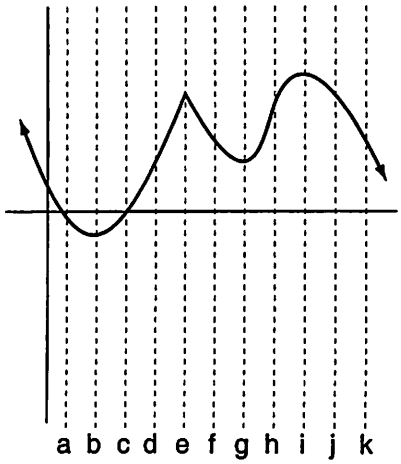
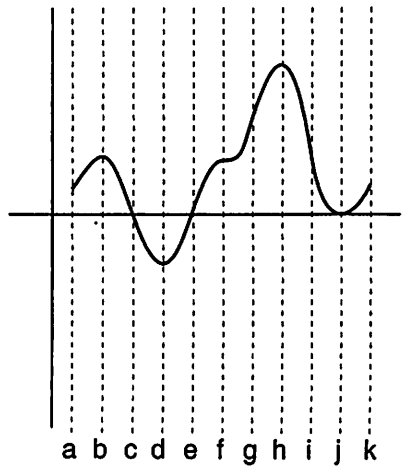
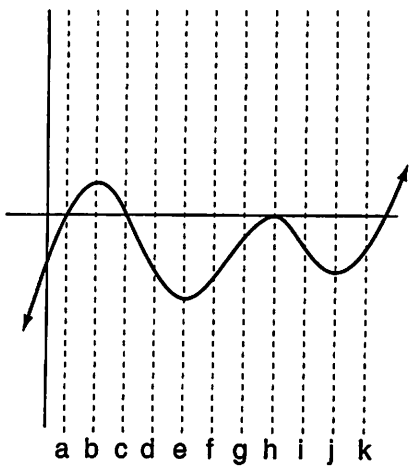


c)





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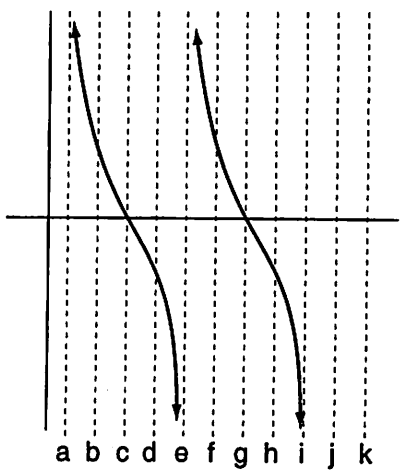

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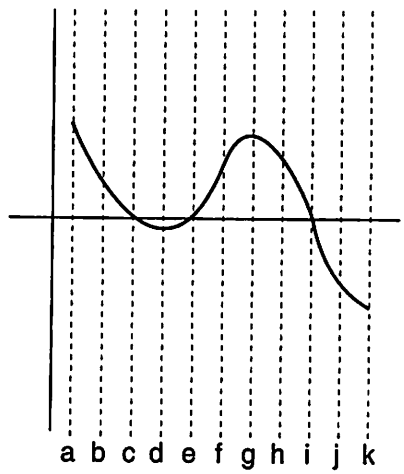
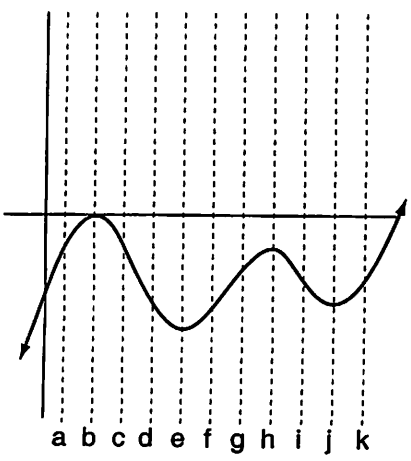
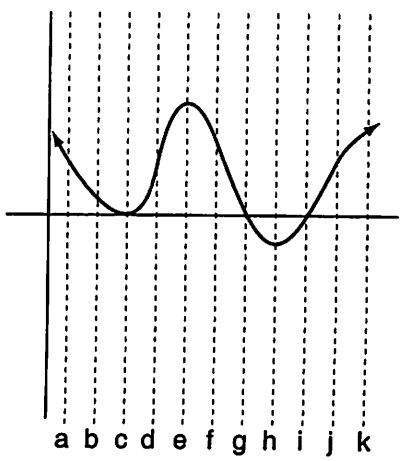


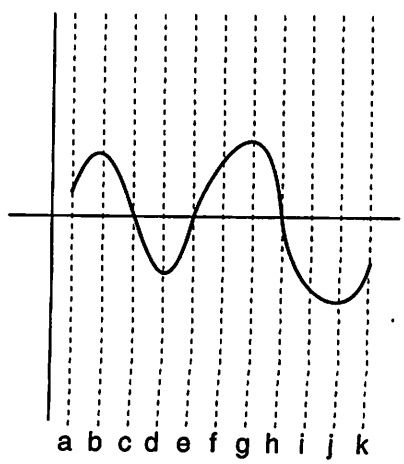
g)



h)



i)

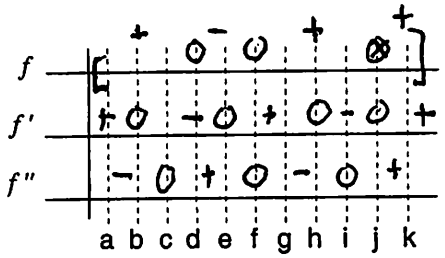


I continued ...

○ = a zero ⊗= Non Participating Point ×= DNE at that point

KEY

a)



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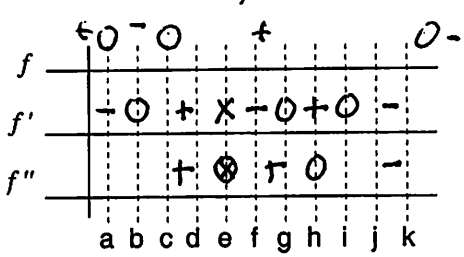

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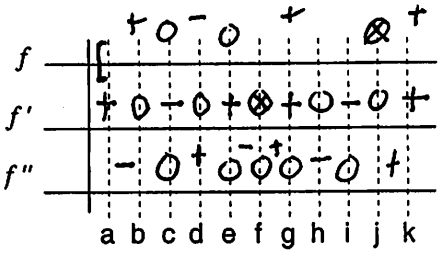
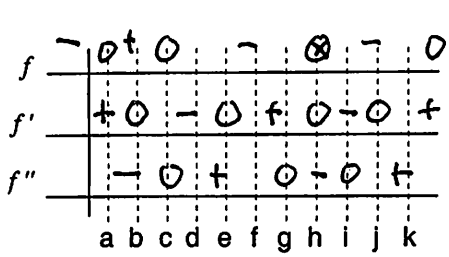


d)



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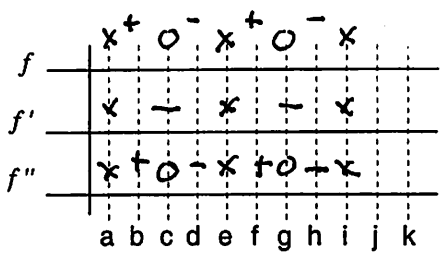

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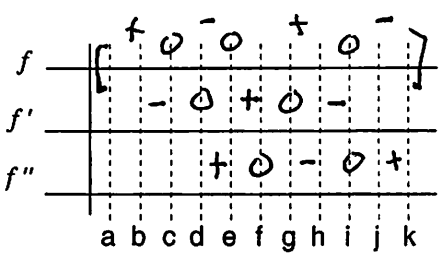

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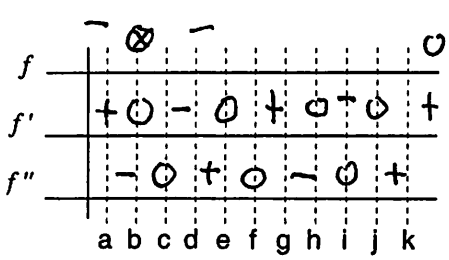

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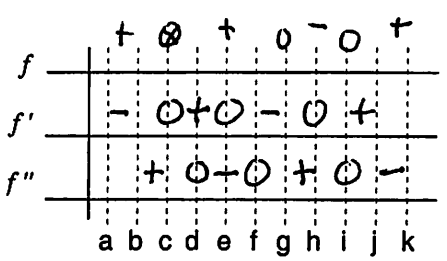

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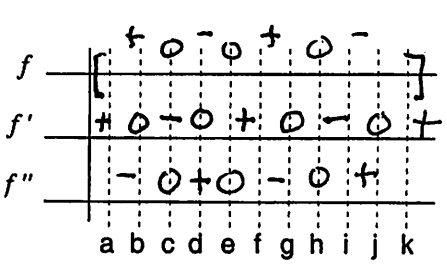
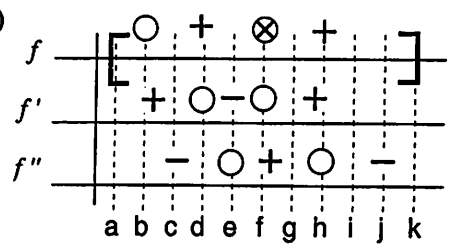


II continued...

a)



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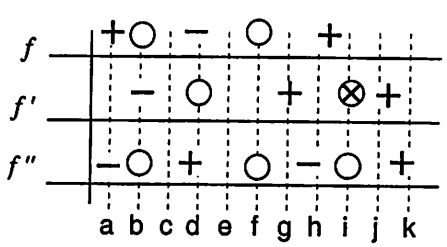

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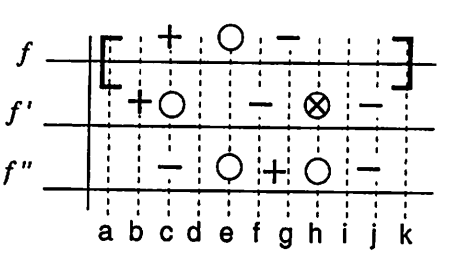

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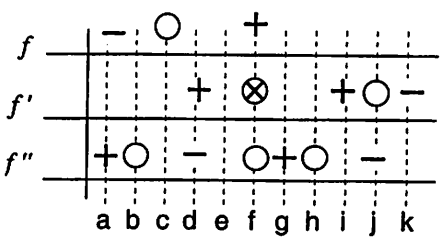

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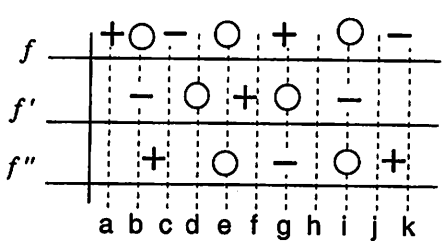

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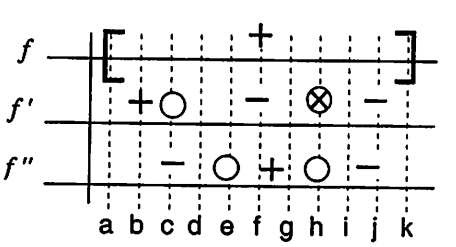

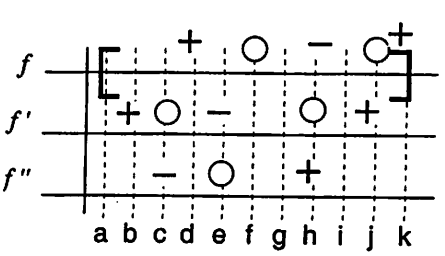

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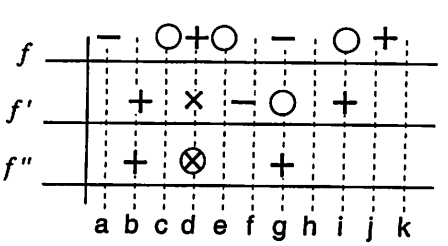

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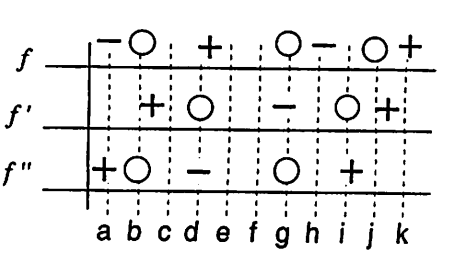


II Use the marked number lines for clues which will allow you to sketch ***only*** the graph of the function, $f(x)$.

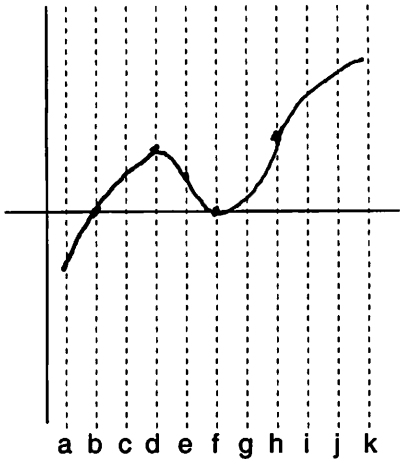
On separate paper, describe all max, min and PI and include proper justification.

KEY

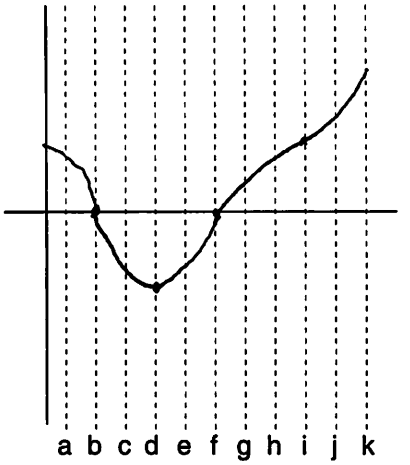
(for example: $(e, f(e))$ max f' test or

$(a, f(a))$ min End Pt. increasing from it.)

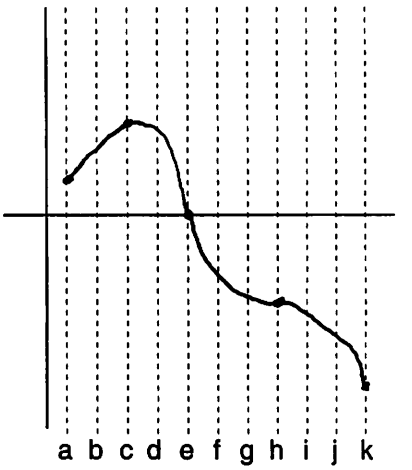
a)



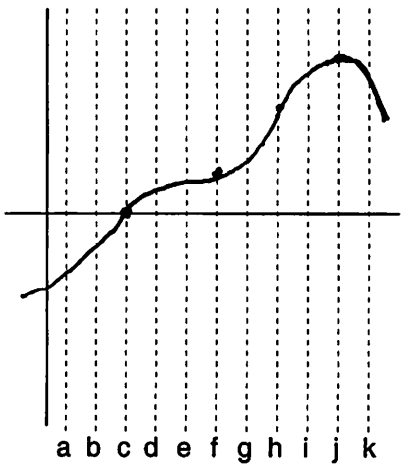
b)



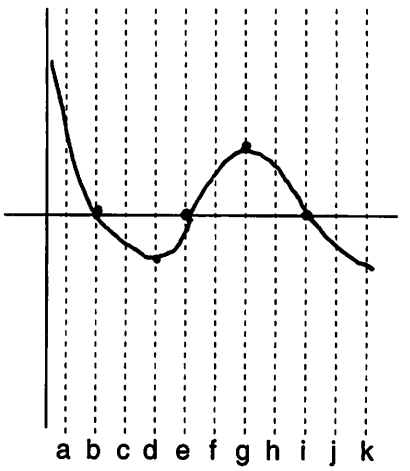
c)



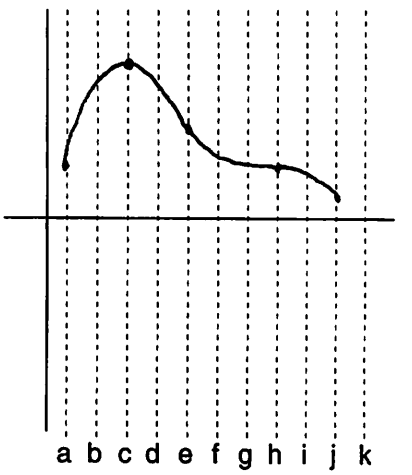
d)



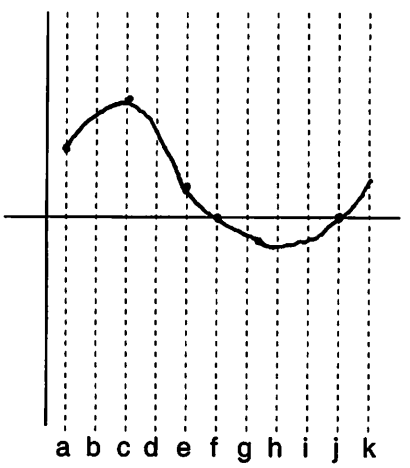
e)



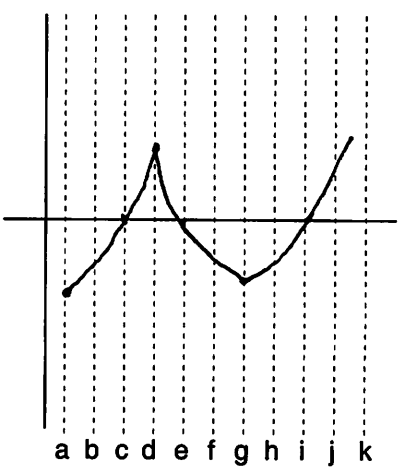
f)



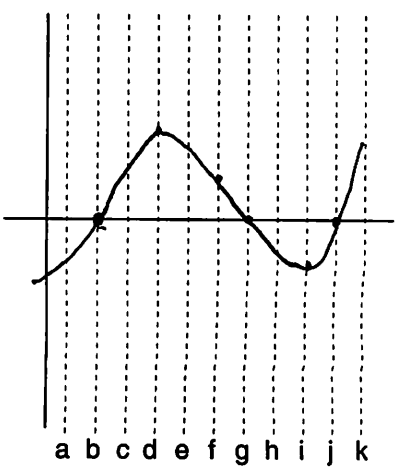
g)



h)



i)



Determine if the following statements are true or false. Explain reasoning. A counter-example is acceptable (you still need to explain your method.)

1) $f(x)=\begin{cases}\dfrac{x^2+2x-3}{x+3} & \text{if } x\neq -3\\ -4 & \text{if } x=-3\end{cases}$ $f(x)$ is continuous at $x=-3$.

2) Let $f(x)$ be a function, D_f $x \in R$.

a) If $f(x)$ is differentiable (has a derivative) at $x=2$, then $f(x)$ must be continuous at $x=2$.

b) If $f(x)$ is continuous at $x=2$, then $f(x)$ must be differentiable at $x=2$.

c) If $f(x)$ is continuous at $x=2$, then $\lim_{x\to 2}\big(f(x)-f(2)\big)=0$.

d) If $f(x)$ is continuous, then $|f(x)|$ is continuous. (T)

e) If $|f(x)|$ is continuous, then $f(x)$ is continuous. (F)

f) If $f(x)$ is continuous, then $f(|x|)$ is continuous. (T)

3) If $f'(x)=g'(x)$, then $f(x)=g(x)$.

4) Let $\lim_{x\to a} f(x)=f\left(\lim_{x\to a} x\right)$.

a) $f(x)$ is continuous.

b) $f(x)$ is differentiable.

Provide a detailed reason for each problem:

5) Define continuity. Give the "Dick & Jane" version and the mathematical version. Prove $f(x)=x^3-8$ is continuous at $x=2$.

6) Write the two versions of the definition of the derivative.

a) Explain how they are different.

b) Explain how they are the same.

c) Show a properly labeled graphical representation for each (as was done in lecture).

d) Tell which version is best suited for which purposes.

e) Using each of the versions, find $f'(2)$ when $f(x)=x^3-8$.

7) Given $f(x)=|x-3|$.

a) Prove $f(x)$ is continuous at $x=3$.

b) Prove $f'(3)$ DNE.

8) Draw the graph of a function which illustrates the situation concerning the definition of continuity:

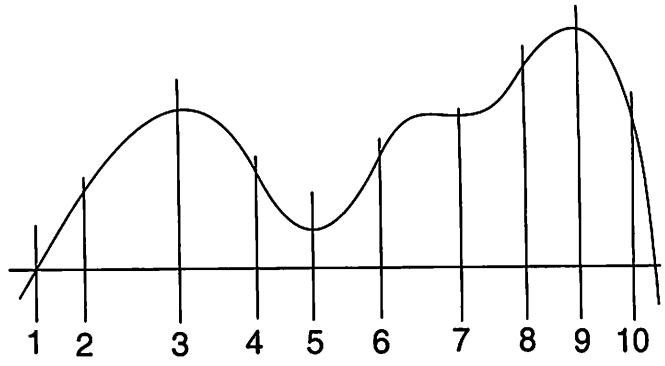
a) Step 1 met. Step 2 met. Step 3 not met.

b) Step 1 not met. Step 2 met.

c) Step 1 not met. Step 2 not met.

d) Step 1 met. Step 2 not met.

9. Use the figure to the right to determine "True" or "False".



a) $f(x)$ changes sign at $x = 1$.

b) $f''(2) = 0$

c) $f'(3) = 0$

d) $f''(4) = 0$

e) $f''(5) = 0$

f) $f''(5) < f''(3)$

g) $f'(7) < 0$

h) $f''(8) = 0$ i) $f(2) < f(7)$ j) $f'(9) = 0$ k) $f'(10) = 10$ l) $f''(10) < 0$

m) $f'(2) < f'(4)$ n) $f'(7) < f''(7)$ o) $f''(2) > f''(4)$

10. Given: $f(x) = \begin{cases} 2x+1 & \text{if } x \varepsilon [-2,0) \\ -x+1 & \text{if } x \varepsilon (0,2) \\ -1 & \text{if } x = 2 \\ 2x-5 & \text{if } x \varepsilon (2,3] \end{cases}$

Note: $x \varepsilon [-2, 0)$ means the same thing as $-2 \le x < 0$

The bracket means "including.." and the parenthesis means "not including"

a) Prove f is or is not continuous at $x = 0$.

b) Prove f is or is not continuous at $x = 2$.

c) Prove $f'(0)$ does or does not exist.

d) Prove $f'(2)$ does or does not exist.

e) Draw the graph of f.

11. Given: $f(x) = 3x^4 - 4x^3$; $D_f x \ \varepsilon \ [-1,2]$

a) Determine all zeros of f.

b) Find $f'(x)$ and $f''(x)$

c) Draw number lines for $f'(x)$ and $f''(x)$.

d) Compute the coordinates of all significant points. (Justify, of course)

e) Graph f.

12. Given: $f(x) = \begin{cases} \dfrac{x^3-1}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$

a) Determine the value of k so f is continuous.

b) Find $f'(1)$

13. Given: $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ k & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$

a) Determine the value of k so f is continuous.

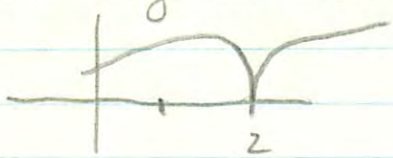
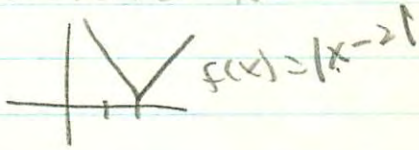
b) Find $f'(1)$

Chapt 3 Ditto

Some interesting pblms.

I A) True $\lim_{x\to -3} = -4$ and $f(-3) = -4$ ∴ continuous

C) False there may be a sharp corner at $x = 2$

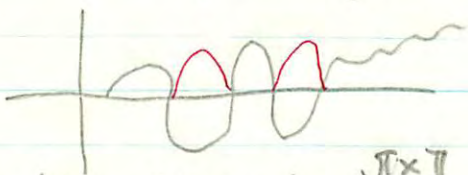
eg:  or 

D) True This is the definition of continuity, eg $f(x) = 2 \ \forall x$. Then $f(x) - f(2) = 2 - 2 = 0$ and then $\lim_{x\to 2}(0) = 0$

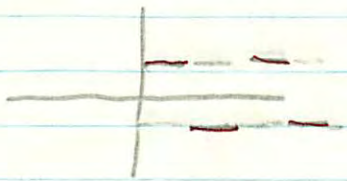
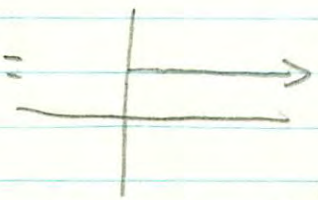
~~B~~ True Since f is continuous $\forall x \in \mathbb{R}$ and IRRATIONALS are $\mathbb{R}$ then $f(x) = x \ \forall x \in \mathbb{R}$ whether x is rational or irrational eg $y = x$

E) FALSE $\lim_{x\to 1}(1+x)^{\frac{1}{x}} = (1+1)^1 = 2$

f) True $|f|$ is simply the neg portion of f reflected across $y=0$ combined with the positive portion of f. If f is continuous then so is $|f|$.



g) FALSE consider $f(x) = (-1)^{[\![x]\!]}$. $x \geq 0$

$f(x) =$  $|f(x)| =$ 

h) FALSE Limit of a fctn is not fct of the limit.

$\lim_{x\to a} x = a$ which is $f(a)$ may or may be limit of fctn. eg $\operatorname{Sgn} x$ $\lim_{x\to 0}|\operatorname{Sgn} x| = 1$ $\lim_{x\to 0} x = 0$

$|\operatorname{Sgn} x| = 0$.

I is false $g = x^2 + 1$, $x^2 = f$ $f' = g'$ but $f \neq g$.

II

A) True $3 - \varepsilon > 0$ $3 + \varepsilon < 0$
B) False $f''(2)$ appears to be Negative. Convex up.
C) True m of tangent appears to be 0.
D) True convex down
E) False convex down over that interval
f) False Sharp corner exists $\therefore$ $F'(3)$ does not.

III

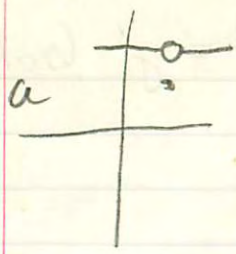
A) a fcn is continuous at a if
① $f(a)$ Exists and.
② $\lim_{x \to a} f(x) = f(a)$

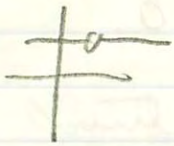
B) $f(x) = 2t^3$
$f(1) = 2(1)^3 = 2$

$\lim_{t \to 1} 2t^3 = 2 \cdot 1^3 = 2$ $\lim_{t \to 1} f(t) = f(1)$

$\therefore$ f is continuous at $t = 1$

C NO. $f(t) = |t|$ is continuous at $t = 0$ but obviously not differentiable at $t = 0$. differentiability guarantees continuity but continuity does not guarantee differentiability.

a 

b) 

c)

d)

Mr. Summary

Use a calculator where *necessary* (as contrasted by use due to *laziness*).

I The limit concept

A) Explain, compare and contrast the following: $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a} f(x)$, $\lim_{x\uparrow a} f(x)$, $\lim_{x\downarrow a} f(x)$

B) Explain what "If there is no reason not to, we just stuff it in." means and why and where do we use it.

C) Explain how to "simplify" limit problems. Discuss various methods *but do not use* any specific examples.

II Explain the effect on ....

A) $f(x)$ when $f'(x)>0$ B) $f(x)$ when $f'(x)<0$ C) $f(x)$ when $f'(x)=0$

D) $f(x)$ when $f''(x)=0$ E) $f(x)$ when $f''(x)>0$ F) $f(x)$ when $f''(x)<0$

G) $f'(x)$ when $f''(x)<0$ H) $f'(x)$ when $f''(x)>0$ I) $f'(x)$ when $f''(x)=0$

III Given: $f(x)=\dfrac{x^2+16}{x-3}$ and $f'(x)=\dfrac{x^2-6x-16}{x^2-6x+9}$; also $g(x)=x+3$; $h(x)=f(x)-g(x)$

A) Using long division divide the denominator of $f(x)$ into the numerator of $f(x)$.

B) Explain why there is a vertical asymptote for $f(x)$ at $x=3$. (Describe the situation as $x\to 3^-$ for example.) Be complete! Remember, this *is* an honors class so do honors level work!

C) Compute (maybe you could make your *programmable* calculator do this for you?)

1) Find $f(8)$, $f(80)$, $f(800)$, $f(8000)$ and $f(-2)$, $f(-20)$, $f(-200)$, $f(-2000)$.
2) Find $h(8)$, $h(80)$, $h(800)$, $h(8000)$ and $h(-2)$, $h(-20)$, $h(-200)$, $h(-2000)$

D) State a conclusion about $f(x)$ as x gets very large (and when x gets very negative). What function does $f(x)$ almost duplicate?

E) Make up number lines for $f(x)$ and $f'(x)$.
1) Specifically state the interval(s) where $f(x)$ is above the x-axis.
2) Specifically state the interval(s) where $f(x)$ is increasing.
3) Are your intervals for (1) and (2) the same? Why or Why not?

F) Determine, identify and justify the coordinates of any extrema of $f(x)$.

G) On a single axis...
1) Draw the line $x=3$ (dotted)
2) Draw the line $y=x+3$.
3) Plot the points from part (F).
4) Draw $f(x)$
5) Does your picture agree with your answers to part E (1) and (2)?

IV Given $f(x)=x^4-4x^3+3$ and $[-1,4]$

A) Find the coordinates of all extrema. (Justify, of course.)

B) Sketch $f(x)$, $f'(x)$, and $f''(x)$ on a single y-axis with 3 different x-axes.

V Given $f(x)$ is continuous at $x=a$. Only one of the following may be false. Determine which one may be false and draw a picture of a function which satisfies all of the others except that one you have picked.

i) $\lim_{x\to a} f(x)$ exists ii) $\lim_{x\to a} f(x)=f(a)$ iii) $f(a)$ and $f'(a)$ exist iv) $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$

VI The derivative of $f(x)=\frac{1}{3}x^4-\frac{1}{5}x^5$ attains its maximum value(s) at what value(s) of x?

VII The line $3x-4y=0$ is tangent in the first quadrant to $y=x^3+k$. Find k.

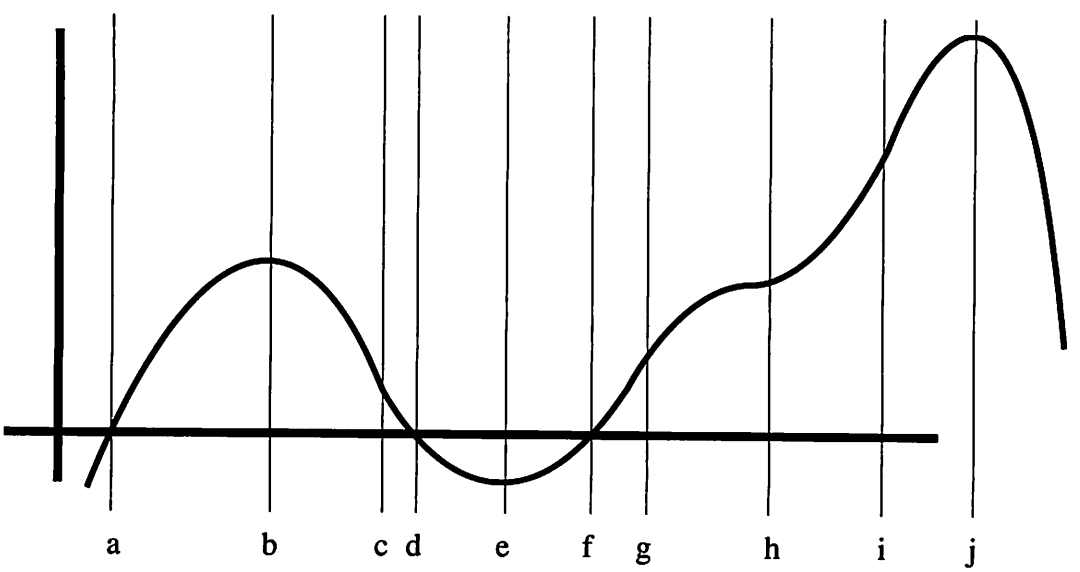
VIII Given $f(x)=3x^5-20x^3$. Sketch. Determine extrema. (Justify, of course)

IX Given $f(x)=x^3-x^2-4x+4$. Find the equation of the tangent line to $f(x)$ at $x=-1$.

X True or False (and explain): $f(x)=5x^4-x^5$ has two inflection points.

XI Using words only, explain the three step process required to prove that a function is continuous at a particular point. Give picture examples of functions which illustrate when one or more of the "parts of the definition of continuity" has been violated.

XII The following picture is the graph of $f'(x)$. Note it is *not* the graph of $f(x)$.
Draw the pictures of $f(x)$ and $f''(x)$.
Specifically state coordinates of significant items. Justify, of course.

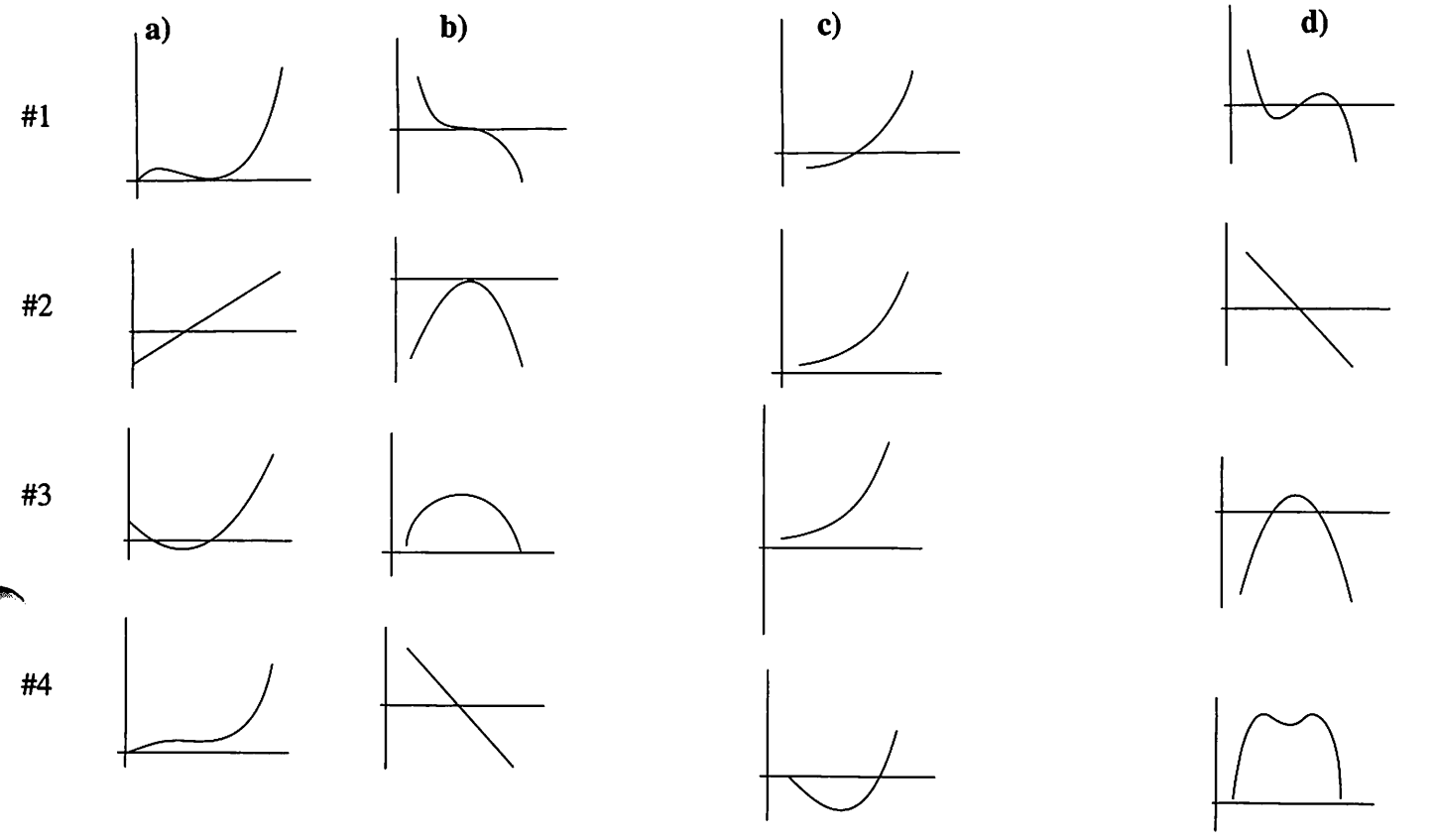


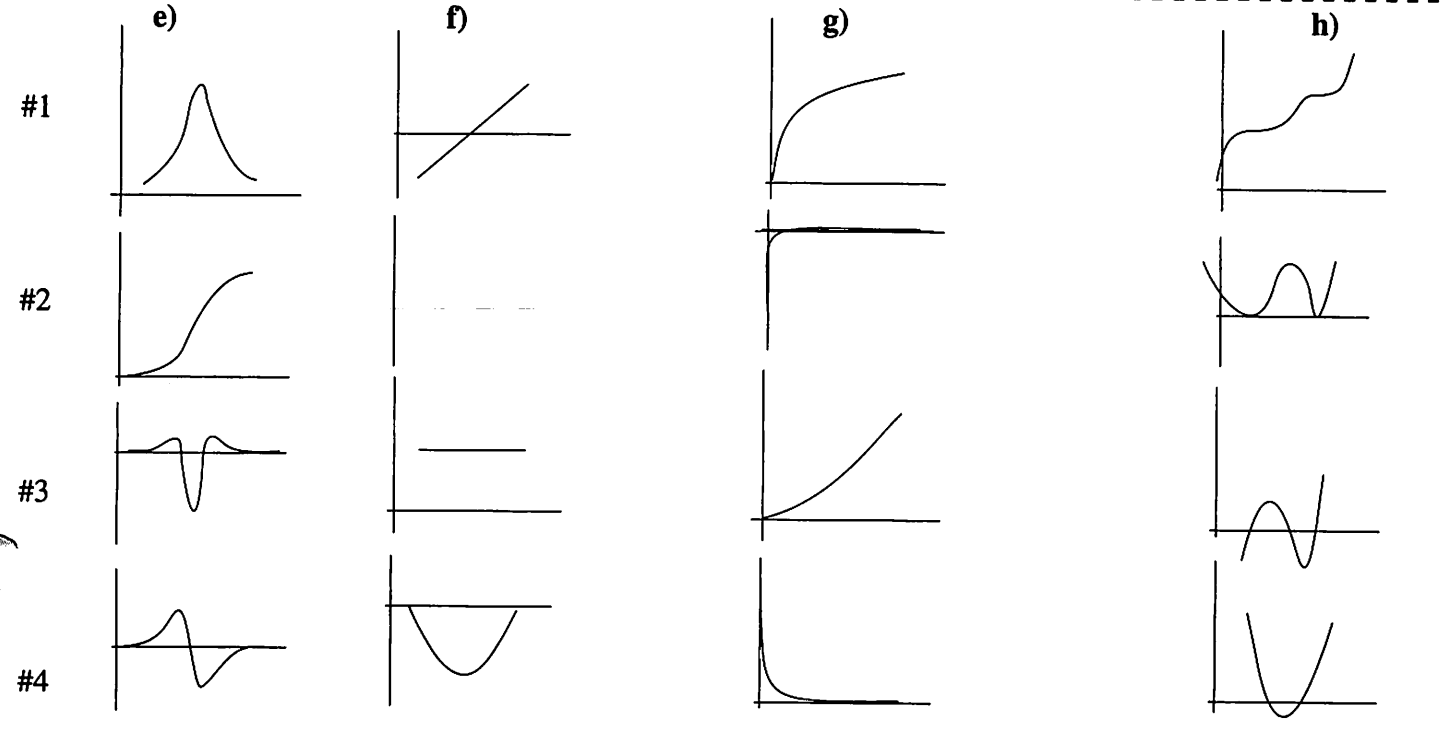
Analysis

J Mumaugh

We want *contenders!* Can you find the *pretender*?

Each of the following columns is headed by the graph of a function $f(x)$. Underneath this graph are three others that are, in random order, the graphs of $f'(x), f''(x)$, and another function. Your job is to determine the pretender and to label the others properly.

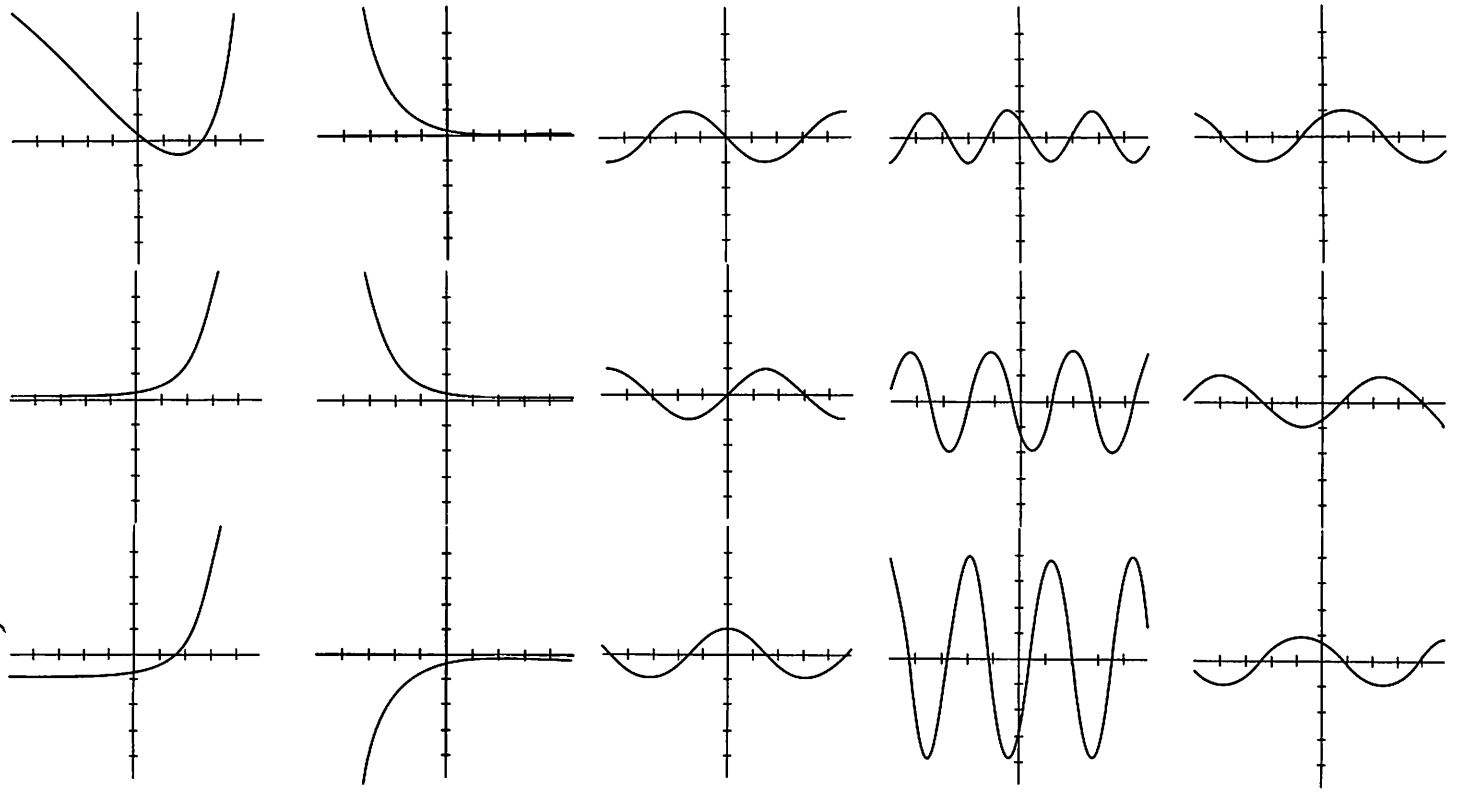




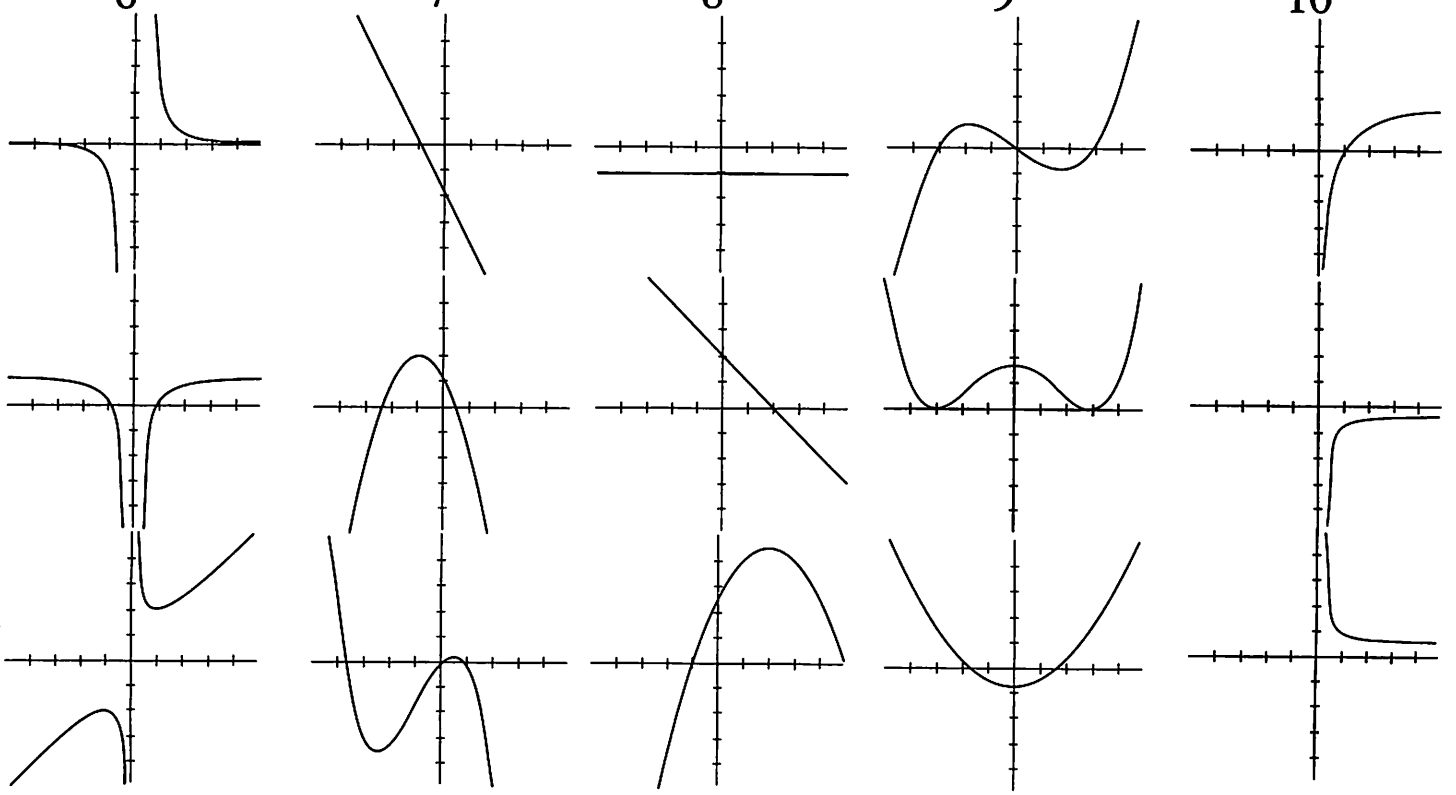
Sherlock Heston Knows his f from his f' and his f''. Do you?

Under each number are 3 axes. The curves, f , f', and f'' are in random order. Your job is to decide which curve is which. Consider problem #1. The first graph is f and it is concave up everywhere so the 2nd curve would be f'' since it is everywhere positive and the third curve changes from negative to positive when f changes from decreasing to increasing so it would be f'. Include similar details on your homework.

1 2 3 4 5



6 7 8 9 10



Honors Analysis Chapter 3 Heavy Duty Quiz Name _____

Open Notes NOT open friends **25 minutes** Period _____ Date _____

Do all work and put answers on this paper.

I	II	III	IV

I What is the effect on:

6 +

a) $F(x)$ and $F'(x)$ when $F''(x) > 0$ — f concave up f' increasing

5 ✓

b) $F(x)$ when $F'(x) = 0$ — max, min or PI

4 0

c) $F(x)$ when $F'(x) < 0$ — f decreases

3 −

d) $F(x)$ when $F'(x) > 0$ — f increases

0,1,2 =

e) $F(x)$ and $F'(x)$ when $F''(x) < 0$ — f concave down f' decreases

f) $F(x)$ and $F'(x)$ when $F''(x) = 0$ — f may have PI, f' max min or PI

II Given $F'(x)$ and $F''(x)$ number lines. Sketch the graph of a function which has the properties described:

a)

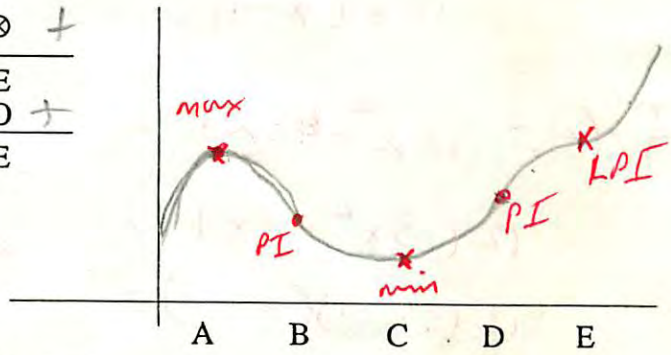
	A		B		C		D		E	
$F'(x)$	+	O		−		O	+	⊗		+
$F''(x)$	−		O		+		O	−	O	+
	A		B		C		D		E	

3 .

2 ✓

1 0 ← 3 wrong graphs

0 −



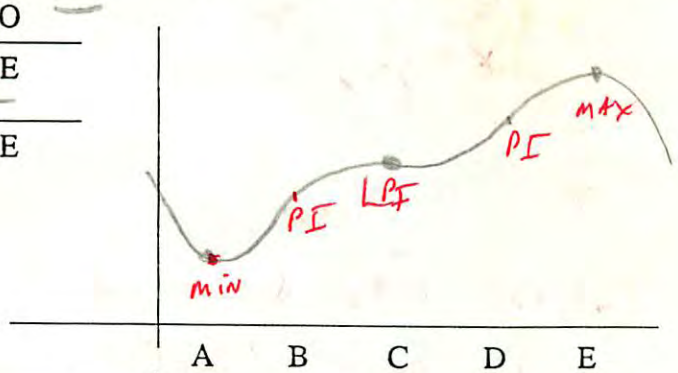
b)

	A		B		C		D		E	
$F'(x)$	−	O		+		⊗		+		O	−
$F''(x)$	+		O	−	O		+	O	−		
	A		B		C		D		E	

~~Nothing~~ =

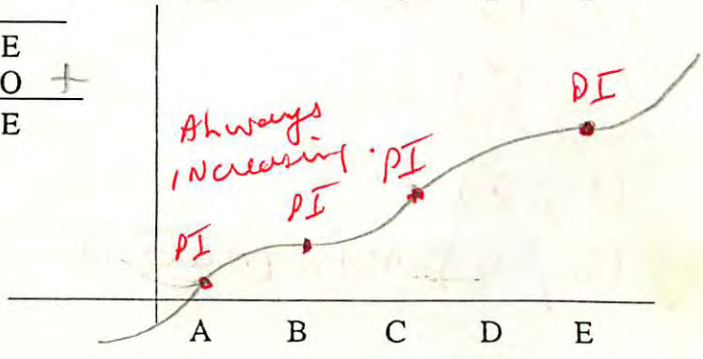
↑ ~~Start~~ from Less

3 wrong; 1 or 2 only tried



c)

	A		B		C		D		E	
$F'(x)$					+					
$F''(x)$	+	O	−	O	+	O	−		O	+
	A		B		C		D		E	



Sections III and IV on back.

III Find the limit: $\lim_{x \to 1}\left(\frac{1}{1-x^3}-\frac{1}{3(1-x)}\right)$

$(1-x)(1+x+x^2)$

$$\lim_{x\to 1}\left(\frac{3-(1+x+x^2)}{3(1-x)(1+x+x^2)}\right)$$

$$\lim_{x\to 1}\frac{3-1-x-x^2}{3(\ )(\ )}$$

$$\lim_{x\to 1}\frac{2-x-x^2}{3(\ )(\ )}$$

$$\lim_{x\to 1}\frac{(2+x)(1-x)}{3(1-x)(1+x+x^2)}$$

$$\frac{3}{3\cdot 3}=\frac{1}{3}$$

Correct +

has limit ∴ every limit is wrong answer ✓

Left off limit and right answer =

and wrong answer =

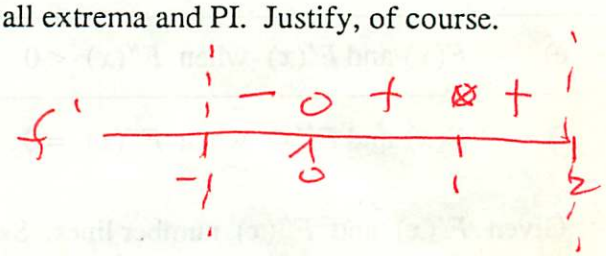
IV Given: $F(x)=3x^4-8x^3+6x^2+1 \quad \forall\ x\ \varepsilon\ [-1,\ 2]$.

Determine (identify as local or global) the *coordianates* of all extrema and PI. Justify, of course.

$$f'(x)=12x^3-24x^2+12x$$

$$12x(x^2-2x+1)$$

$$12x(x-1)^2$$

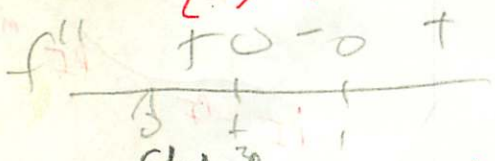


$$f''(x)=36x^2-48x+12$$

$$12(3x^2-4x+1)$$

$$12(3x-1)(x-1)\overset{set}{=}$$

$$x\in\left\{\frac{1}{3},\ 1\right\}$$



F	3	−8	6	0	1
−1	3	−11	17	−17	18
0					1
1	3	−5	1	1	2
2	3	−2	2	4	9
$\frac{1}{3}$	3	−7	$\frac{11}{3}$	$\frac{11}{9}$	$\frac{38}{27}$

Global MAX $(-1, 18)$ def'n of f' test

Global min $(0, 1)$

$\left(\frac{1}{3}, \frac{38}{27}\right)$ PI

$(1, 2)$ LPI

Local MAX INC to it $(2, 9)$

5	+
4	✓
3	0
2	−
0 1	=

To be correct part must have
① coordinate
② identify
③ justify.

Heavy Duty Quiz Name _____

Open Notes NOT open friends **25 minutes** Period _____ Date _____

Do all work and put answers on this paper.

I	**II**	**III**	**IV**

I What is the effect on:

a) $f(x)$ and $f'(x)$ when $f''(x) > 0$

b) $f(x)$ when $f'(x) = 0$

c) $f(x)$ when $f'(x) < 0$

d) $f(x)$ when $f'(x) > 0$

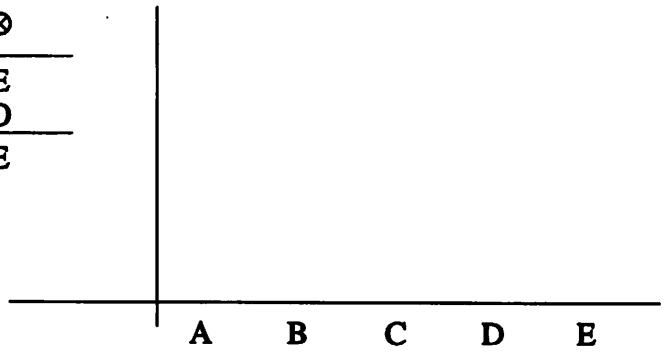
e) $f(x)$ and $f'(x)$ when $f''(x) < 0$

f) $f(x)$ and $f'(x)$ when $f''(x) = 0$

II Given $f'(x)$ and $f''(x)$ number lines. Sketch the graph of a function which has the properties described:

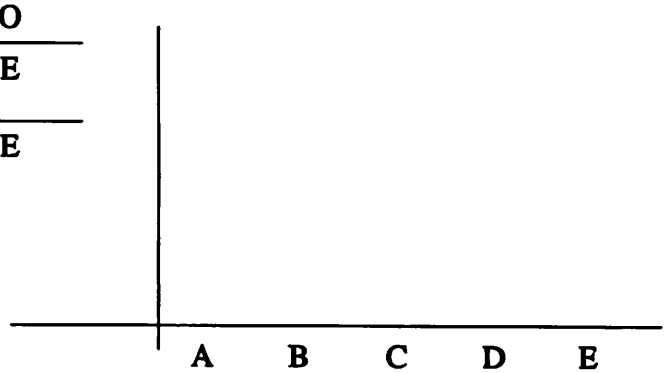
a)

$f'(x)$	O		O	+	⊗
	A	B	C	D	E
$f''(x)$	−	O		O	O
	A	B	C	D	E



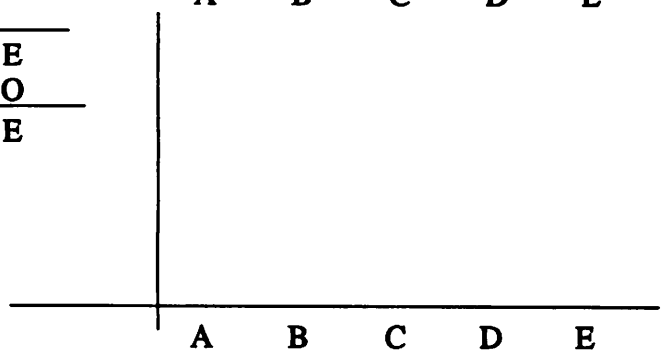
b)

$f'(x)$	− O		⊗		O
	A	B	C	D	E
$f''(x)$	+	O	O	O	
	A	B	C	D	E



c)

$f'(x)$			+		
	A	B	C	D	E
$f''(x)$	O	O	O	−	O
	A	B	C	D	E



Sections III and IV on back.

Honors Analysis Chapter 3 Heavy Duty Quiz Name _____

Open Notes NOT open friends **25 minutes** Period _____ Date _____

Do all work and put answers on this paper.

I	**II**	**III**	**IV**

I What is the effect on:

a) $f(x)$ and $f'(x)$ when $f''(x) > 0$

b) $f(x)$ when $f'(x) = 0$

c) $f(x)$ when $f'(x) < 0$

d) $f(x)$ when $f'(x) > 0$

e) $f(x)$ and $f'(x)$ when $f''(x) < 0$

f) $f(x)$ and $f'(x)$ when $f''(x) = 0$

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a)

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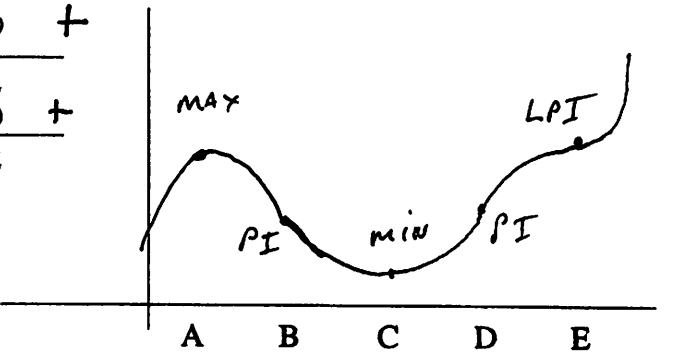
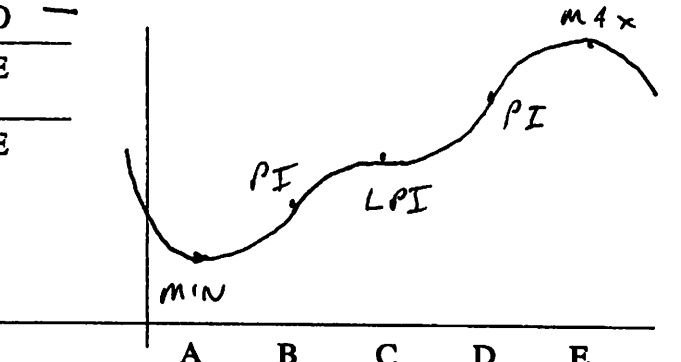
$f'(x)$	+	O		−		O	+	⊗		+
$f''(x)$	−		O		+		O	−	O	+



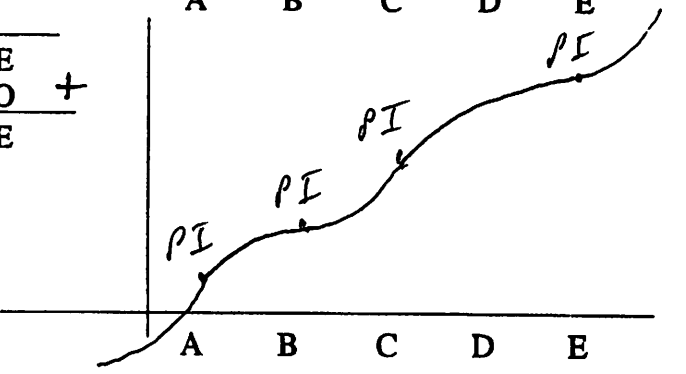
b)

	A		B		C		D		E
$f'(x)$	−	O		+		⊗		+	O	−
$f''(x)$	+		O	−	O	+	O	−		



c)

	A		B		C		D		E	
$f'(x)$						+				
$f''(x)$	+	O	−	O	+	O	−	O	+	



Sections III and IV on back.

III Find the limit: $\lim_{x \to 1} \left(\frac{1}{1-x^3} - \frac{1}{3(1-x)} \right)$

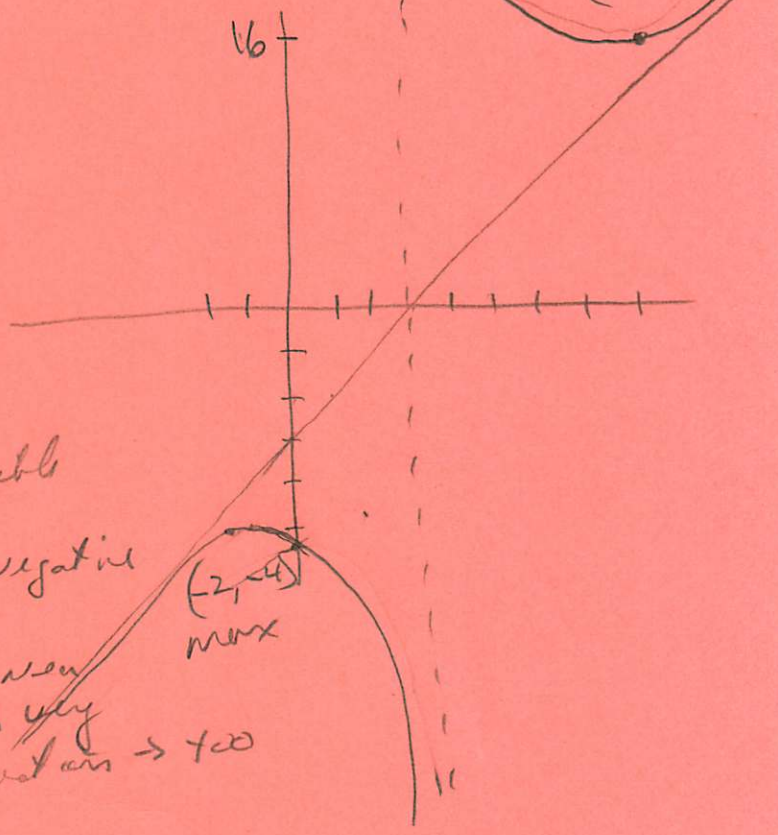
IV Given: $f(x) = 3x^4 - 8x^3 + 6x^2 + 1 \quad \forall \; x \; \varepsilon \; [-1, \; 2]$.
Determine (identify as local or global) the *coordinates* of all extrema and PI. Justify, of course.

MR Summary

III $f(x) = \dfrac{x^2+16}{x-3} = x+3+\dfrac{25}{x-3}$

a)
$$\begin{array}{r|l} & x+3 \\ x-3 & x^2+16 \\ & x^2-3x \\ \hline & 3x \\ & 3x-9 \\ \hline & 25 \end{array}$$

1 2 3 4 5 6



b) There is a vertical asymptote because $x=3$ is a non-removable Domain Restriction.

$\lim_{x\to 3^-} f(x) = \frac{+}{-} < 0$ but very negative

$\lim_{x\to 3^+} f(x) = \frac{+}{+} > 0$ as x gets very near 3, Bottom is very small so $f \to +\infty$

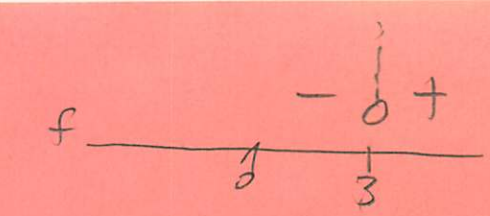
c)

$f(8)=16$	$f(80)=83.3$	$f(800)=803,031$	$f(8000)=8003,003$
$g(8)=11$	$=83$	$=803$	$=8003$
$h(8)=5$	$=.325$	$=.031$	$=.0031$
$f(-2)=-4$	$f(-20)=-18.09$	$f(-200)=-197,123$	$f(-2000)=-199,012$
$g(-2)=1$	$g(-20)=-17$	$=-197$	$=-199$
$h(-2)=-5$	$h(-20)=-1.09$	$=-.123$	$=-.0125$

D) as $x \to \pm\infty$ f and g are almost the same. Since their difference becomes zero.

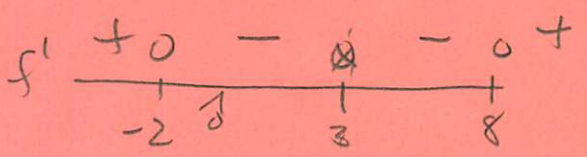
mr Summary

E)



$f(x) = \dfrac{x^2+16}{x-3}$

$f'(x) = \dfrac{(x+2)(x-8)}{(x-3)^2}$



f is > 0 when $x > 3$

f is increasing when $x < -2$ or $x > 8$

intervals are not the same. They didn't need to be. a fctn can be positive and not be increasing (for example)

F) max at $(-2, -4)$ f' test

min at $(8, 16)$ f' test.

G) on Previous page.

IV $f(x) = x^4 - 4x^3 + 3 \quad [-1, 4]$

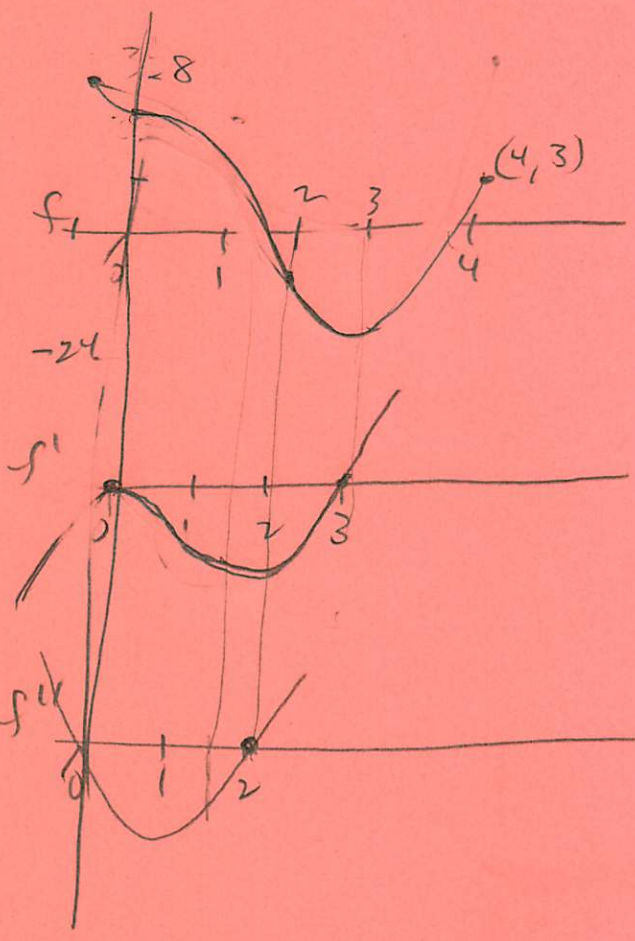
$f(-1) =$

$f(1) = 1 - 4 + 3 = 0$ $\quad f(3) = -24$

$f(4) = 3$ $\quad f(-1) = 8$

$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3)$



$f''(x) = 12x^2 - 24x$

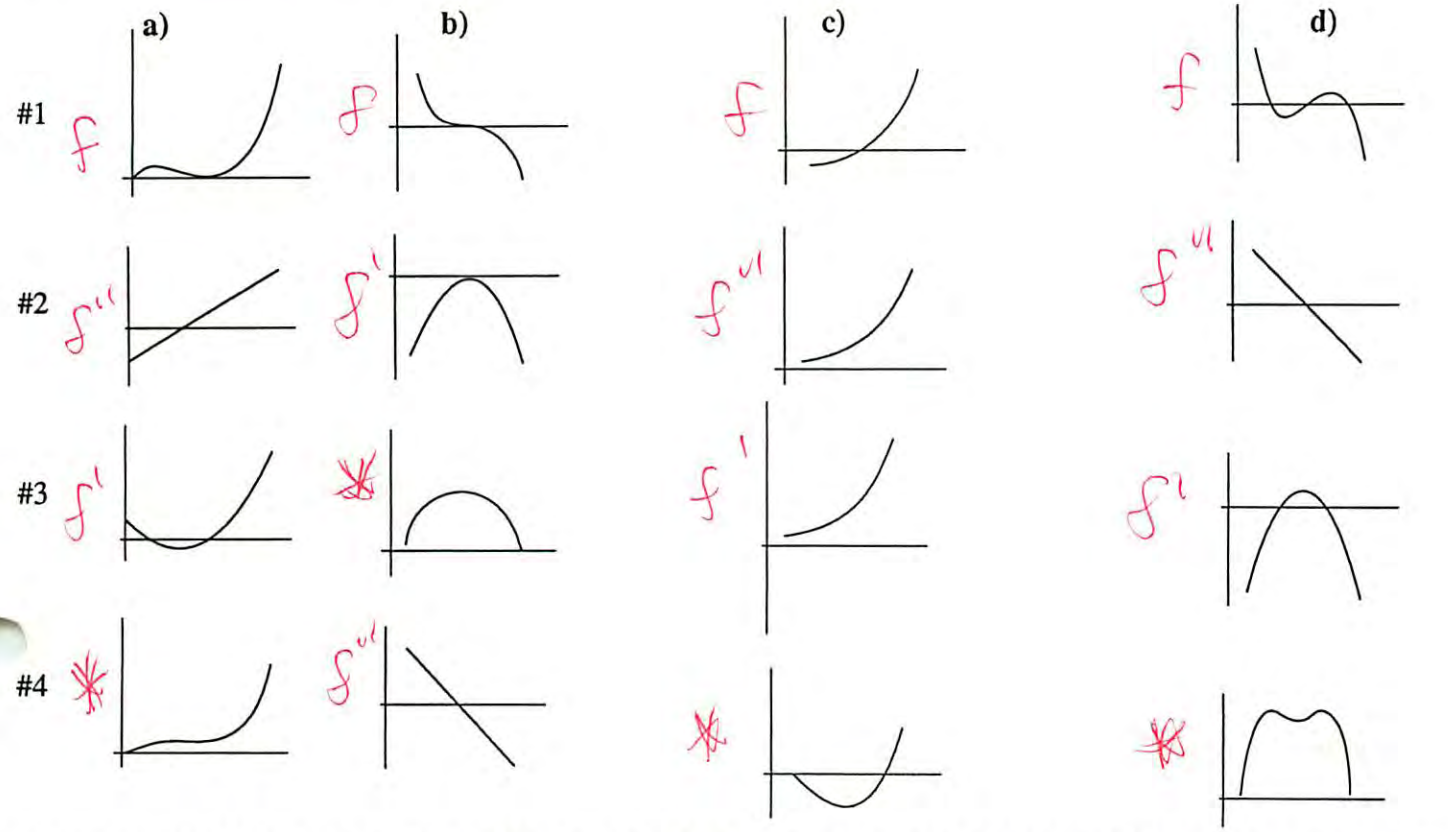
$12x(x-2)$

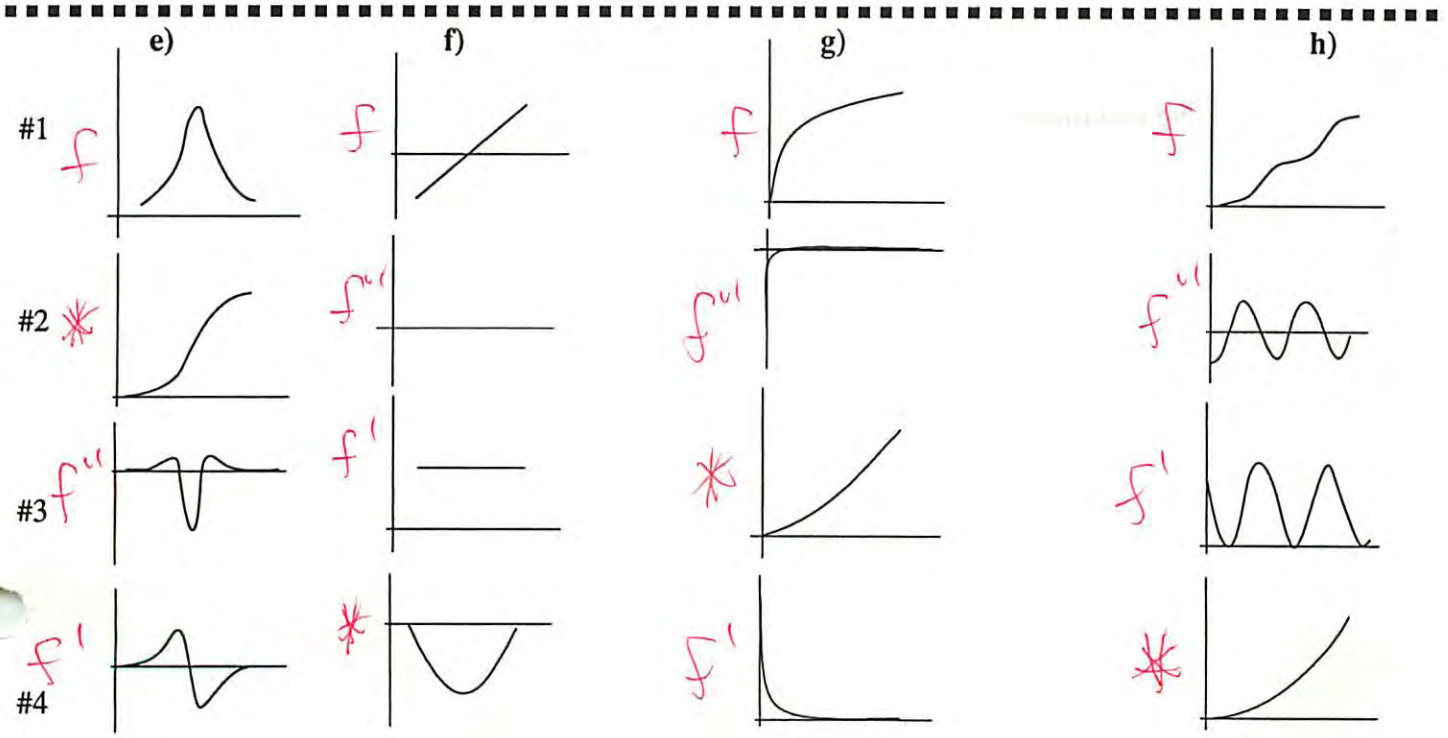
Analysis

J Mumaugh

We want *contenders!* Can you find the *pretender*?

Each of the following columns is headed by the graph of a function $f(x)$. Underneath this graph are three others that are, in random order, the graphs of $f'(x), f''(x)$, and another function. Your job is to determine the pretender and to label the others properly.

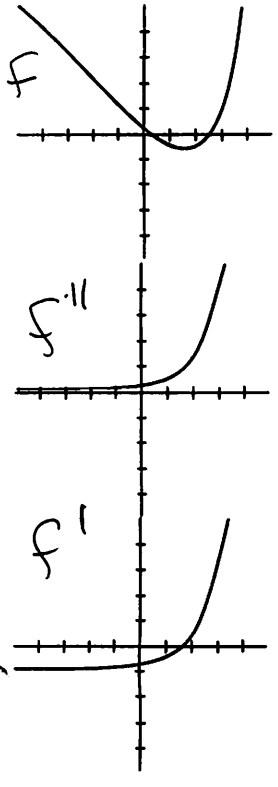




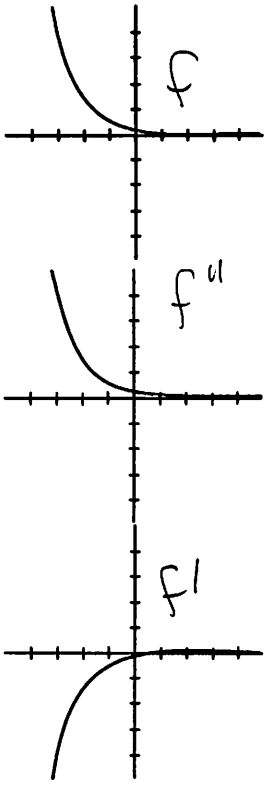
Sherlock Heston Knows his f from his f' and his f''. Do you?

Under each number are 3 axes. The curves, f , f', and f'' are in random order. Your job is to decide which curve is which. Consider problem #1. The first graph is f and it is concave up everywhere so the 2nd curve would be f'' since it is everywhere positive and the third curve changes from negative to positive when f changes from decreasing to increasing so it would be f'. Include similar details on your homework.

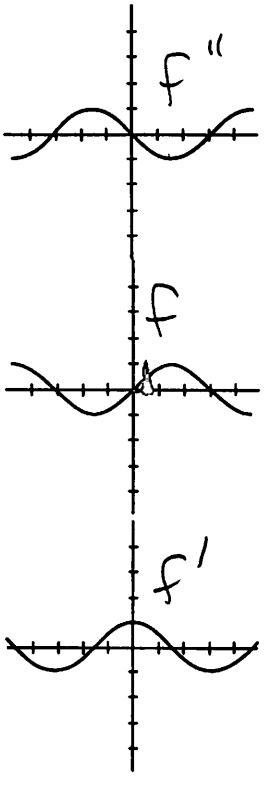
1



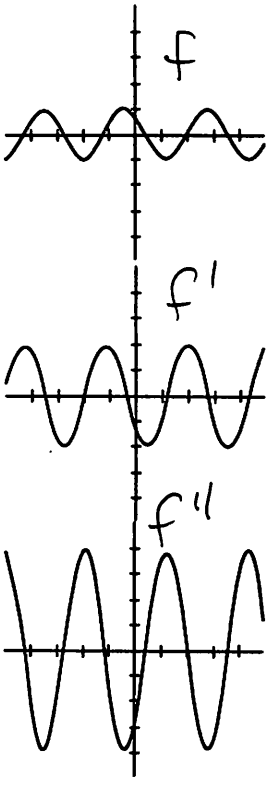
2



3



4



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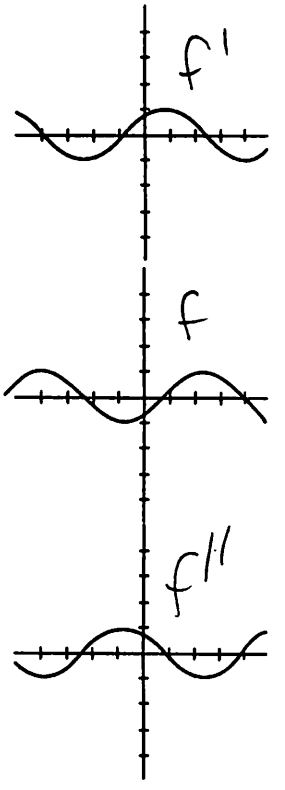
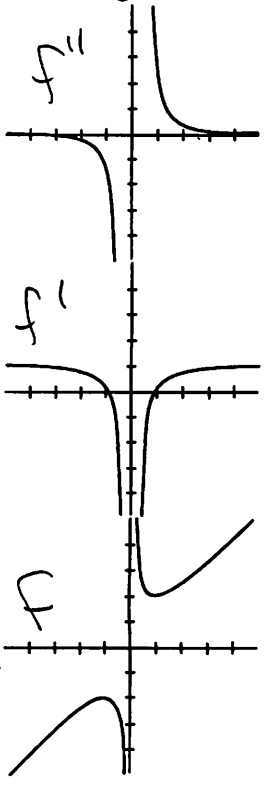


6



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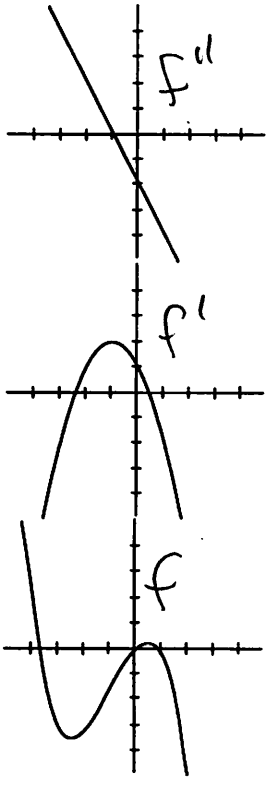

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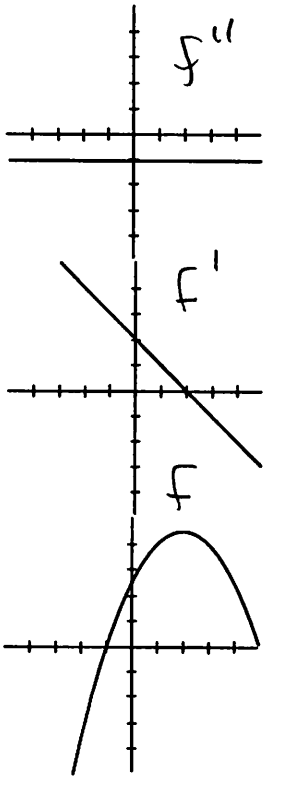

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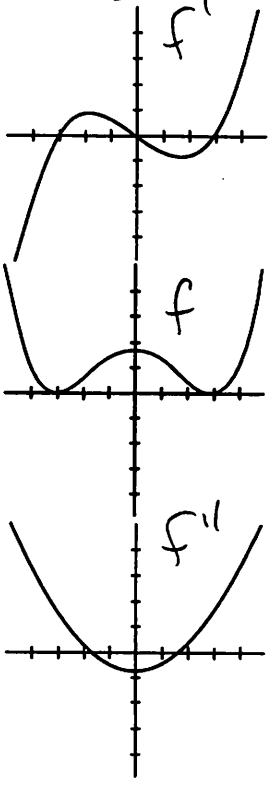

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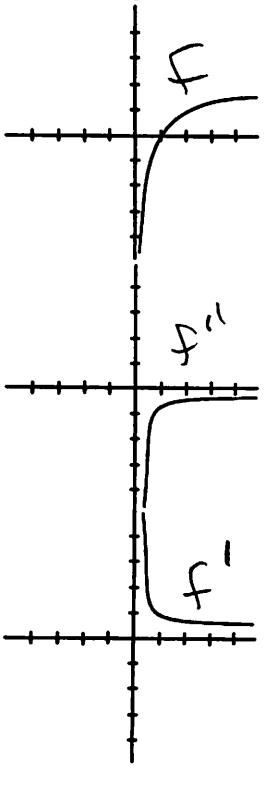
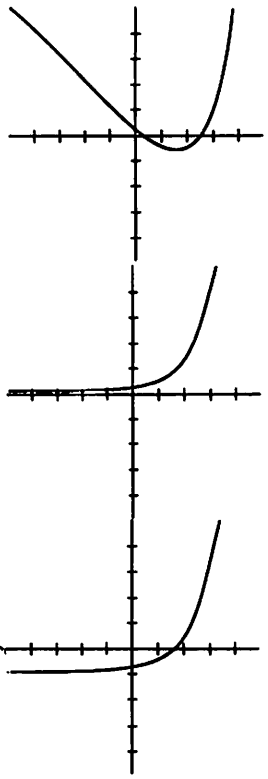


Sherlock Heston Knows his f from his f' and his f''. Do you?

Under each number are 3 axes. The curves, f , f', and f'' are in random order. Your job is to decide which curve is which. Consider problem #1. The first graph is f and it is concave up everywhere so the 2nd curve would be f'' since it is everywhere positive and the third curve changes from negative to positive when f changes from decreasing to increasing so it would be f'. Include similar details on your homework.

1



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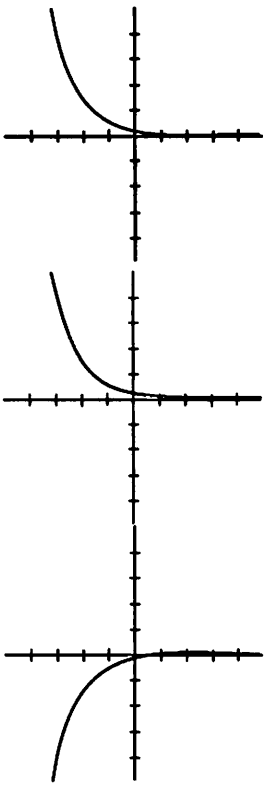

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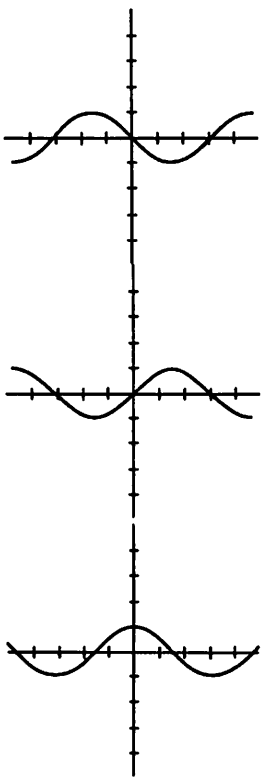

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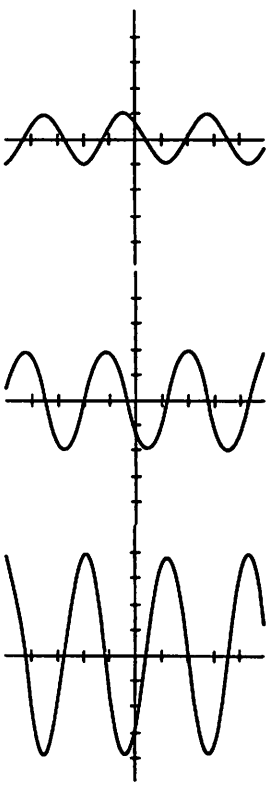

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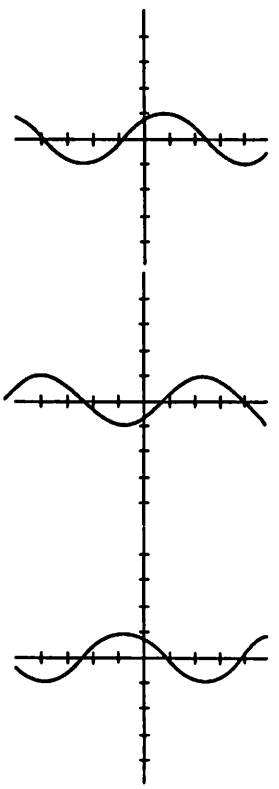

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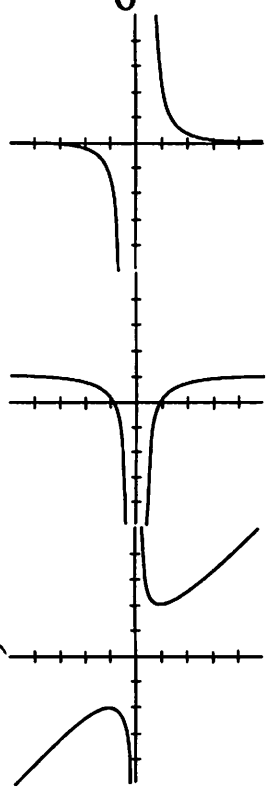

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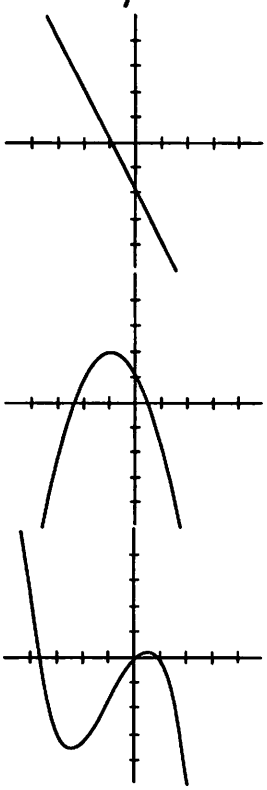

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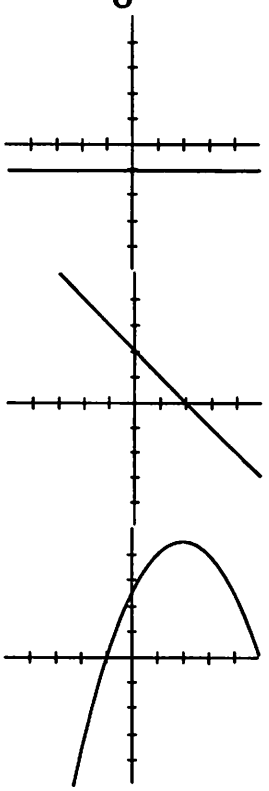

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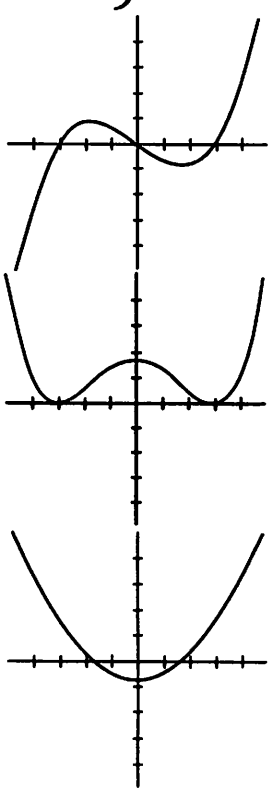

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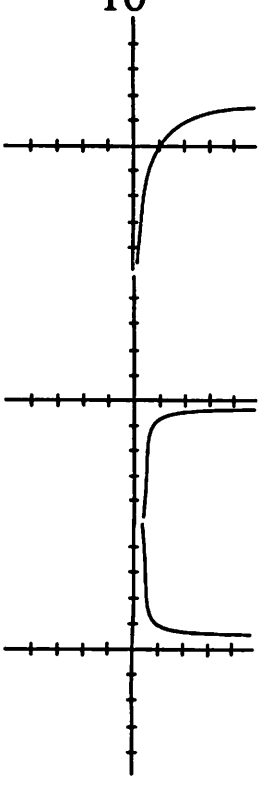
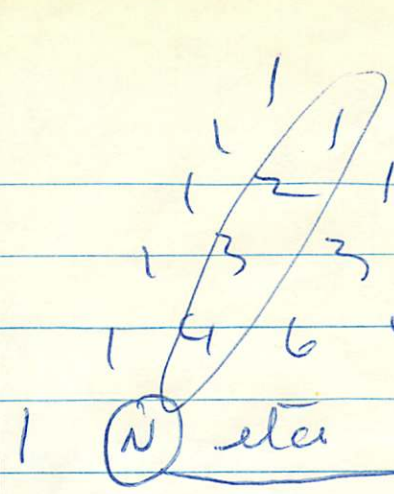


The pictures are placed in the following format:

1A	3A	4C	5A	6B
1B	3C	4A	5B	6A
1C	3B	4B	5C	6C

2C	7C	8B	9B	10A
2B	7B	8C	9A	10C
2A	7A	8A	9C	10B



$\Leftarrow$ Exponent column.

$\therefore (x+h)^N =$

$x^N + Nx^{N-1}h + \text{etc.}$

$$\frac{(x+h)^N - x^N}{h} = \frac{x^N + Nx^{N-1}h + \text{etc} - x^N}{h}$$

$$\frac{Nx^{N-1}h + \text{etc}}{h} =$$

$$\lim_{h \to 0} \; Nx^{N-1} + \underbrace{(N-1)x^{N-2}h + \cdots}_{0}$$

$\therefore$

$$\frac{d(x^N)}{dx} = Nx^{N-1}$$

(1) $\lim_{x \to 1} x^2 + 2x$

$\lim_{x \to 1} x \cdot x + 2\lim_{x \to 1} x = \left(\lim_{x \to 1} x\right)\left(\lim_{x \to 1} x\right) + 2\lim_{x \to 1} x = 1 + 2 = 3$

(2) $\lim_{x \to 1} (3x - 2x^2) = 1$

(3) $\lim_{x \to 0} (x^3 + 4) = 4$

(4) $\lim_{x \to 0} (2 - x^3) = 2$

(5) $\lim_{x \to -1} (x^2 + x - 2) = -2$

(6) $\lim_{x \to -2} (x^2 + x - 2) = 0$

(7) $\lim_{x \to 2} \frac{x + 2}{x^3} = \frac{1}{2}$

(8) $\lim_{x \to -1} \frac{x + 1}{x^2 + 1} = 0$

(9) $\lim_{x \to 1} \frac{x^2 - 2x}{x^2 + 2x} = -\frac{1}{3}$

$\frac{x(x+1)}{x}$

(10) $\lim_{x \to 2} \frac{x^3 + x^2}{x^2 + 2} = \frac{12}{6} = 2$

(11) $\lim_{x \to 1} \frac{x - 1}{x^2 - 1} = \frac{1}{x + 1} = \frac{1}{2}$

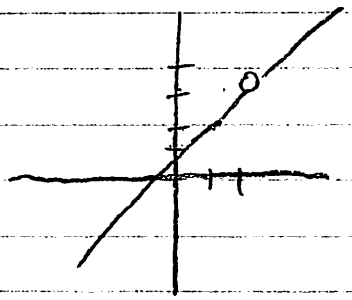
(12) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2} = \therefore \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{4}{3}$

$\frac{x-4}{x^2 - x - 2}$

$x - 2$

1st Lecture
Just before Handing out Limit ditto.

Given $f(x) = \dfrac{(x+1)(x-2)}{(x-2)}$



Lets assume $x < 2$.
as x gets bigger f(x) closer and closer to 3.

assume $x > 2$. as x gets smaller
Both f(x) get closer to 3.

Because of this we would say "The limit as x goes to 2 from below and the limit as x goes to 2 from above is 3, ∴ the limit of f(x) is 3." It is written:

$$\lim_{x \to 2} f(x) = 3$$

This was easy to see via the graph, but we shouldn't have to draw every fctn in order to determine its limit.

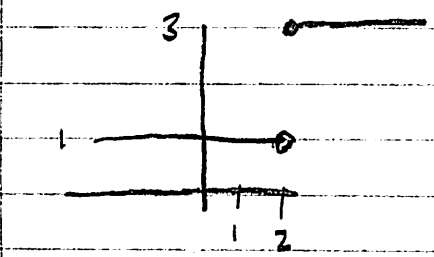
Consider another fctn $g(x) = x+1$. Now, G is exactly like f in all but one place. I know this because I simply did something to f(x) which didn't really change its value — ie I reduced it. That one place is in the Domain.

Since g(x) is so nearly the same as f(x) I can say that whatever g does near $x=2$, f would also Do. Since G has No Domain Restriction at $x=2$ I can say $\lim_{x \to 2} x+1 = 3$. Since I just found $g(2) = 3$

The key is: change the given fctn into another fctn which "acts" like the original, and then take the limit.

Lets go back to our original graph. Notice I said "as x goes to 2 from below" and "as x goes to 2 from above". Both limits are required to make the general statement that "the limit exists"

Contrast



what is the limit as $x \to 2^-$? 1

as $x \to 2^+$? 3

Since $\lim_{x \to 2^-} f(x) = 2$ and

$\lim_{x \to 2^+} f(x) = 3$

we say $\lim_{x \to 2} f(x)$ does not exist.

If we were asked simply to find $\lim_{x \to 2^-} f(x)$ we can say it is 1.

Notice: in both examples, I was not concerned with the fact that the fctn was not defined at the point!

Definition of continuous fctn:

Let $a \in D_f$. Then f is continuous at a iff

$$\lim_{x \to a} f(x) = f(a)$$

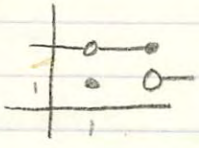
or,

f(x) is continuous if:

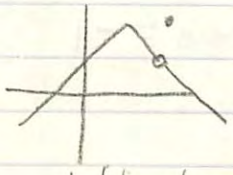
① f is defined at a. AND

② $\lim_{x \to a} f(x) = f(a)$

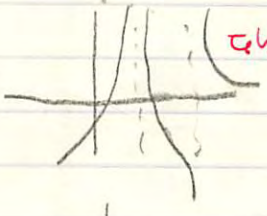
Eg. Pg 119 oral ex.

#2 

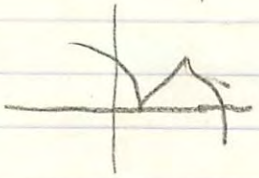
FTN is not continuous, since $\lim_{x \to 1} f(x) = 2$ and $f(1) = 1$ i.e. $\lim_{x \to a} f(x) \neq f(a)$

#3 

Fctn not continuous $\lim_{x \to a} f(x) \neq f(a)$

#4 

Not continuous however it could be said to be continuous (over its domain) for each $a \in D_f$.

#5 

continuous Notice sharp corners.

#6 TRUE

#7

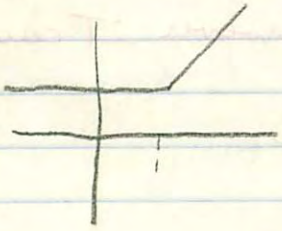
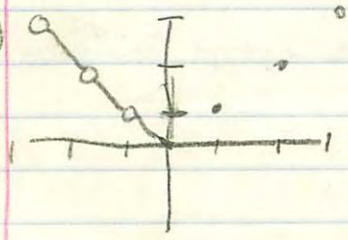
Pg 119

#7 True this is using the definition of continuous fcts

#8 False $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} x+1 = 2$

however $f(1) = \frac{1^2-1}{1-1} = \frac{0}{0}$ it does not name a number.

Written Exercises. Pg 119.

(1) continuous poly fctn
(2) continuous poly fctn
(3) continuous " "
(4) continuous " "
(5) Not continuous at $x=1$
(6) Not continuous at $x=1$
(7) Not continuous
$\lim_{x\to -1^-} f(x) = 1$ $\lim_{x\to -1^+} f(x) = -1$
(8)  continuous
(9)  Not continuous

Book did wrong.

Oral ex.

they are asking what is f' at -2 which means what is the slope of the tangent line at $x=-2$. etc.

(1) $f'(-2) \Rightarrow 2$
(2) $f'(1) \Rightarrow 0$
(3) $f'(3) \Rightarrow -1$
(4) $f'(x)=0$ when $x=1$ and when $x=4$ is change of direction
(5) No "sharp corner."
(6) Positive, increasing
(7) Neg - decreasing
(8) False it could be tangent or it could go directly through the curve
(9) Not necessarily. using the given diagram the tangent at $f(3)$ appears to intersect a second time at $\approx (4, -1)$.

Section 126–130

Given $f(x) = x^3 + 3x^2 - 9x - 2$

A find $f'(x)$

B Evaluate both at $a = -2$

C find equation of tangent at a

D) find coordinates, if any, where tangent is horizontal.

A) $f'(x) = 3x^2 + 6x - 9$

B)
$$\begin{array}{r|cccc} & 1 & 3 & -9 & -2 \\ -2 & & 1 & 1 & -11 \\ \hline & 1 & 1 & -11 & 20 = f(-2) \end{array}$$

$$\begin{array}{r|ccc} -2 & 1 & -1 & -9 = f'(-2) \end{array}$$

C) $f(-2) = 20 \Rightarrow (-2, 20)$

$f'(-2) = -9 = m$

$y - 20 = -9(x + 2)$

$y - 20 = -9x - 18$

$y = -9x + 2$

D) $3x^2 + 6x - 9 \overset{\text{set}}{=} 0 = f'(x)$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1)$

$x = -3 \quad x = 1$

$f(x) \Rightarrow$
$$\begin{array}{r|cccc} & 1 & 3 & -9 & -2 \\ 1 & & 1 & 4 & -5 \\ \hline & 1 & 4 & -5 & -7 \end{array}$$

$$\begin{array}{r|cccc} -3 & 1 & 0 & -9 & 25 \end{array}$$

tangent horizontal at $(1, -7)$ $(-3, 25)$

Discuss Ordinex.

For homework do parts A → D as per instruction

Usual Substitution is Sufficient

① $f(x) = x^2+2x+3 \quad a=-2$

$f'(x) = 2x+2 = 2(x+1)$

	1	2	3	
-2	1	0	3	$= f(-2)$
-2	1	-2		$= f'(-2)$

② $f(x) = 2+3x-x^2 \quad a = 4$

$f'(x) = 3-2x$

	-1	3	2	
4	-1	-1	-2	$= f(4)$
4	-1	-5		$= f'(4)$

③ $f(x) = x^3+3x^2-9x-2 \quad a=2$

$f'(x) = 3x^2+6x-9$

	1	3	-9	-2	
2	1	5	1	0	$= f(2)$
2	1	7	15		$= f'(2)$

④ $f(x) = x^3-3x^2+6x-8 \quad a=3$

$f'(x) = 3x^2-6x+6$

	1	-3	6	-8	
3	1	0	6	10	$= f(3)$
3	1	3	15		$= f'(3)$

(5) $f(x) = x^4 + 4x^3 + 4x^2 - 7$ $\quad a = 1$

$f'(x) = 4x^3 + 12x^2 + 8x$

	1	4	4	0	−7
1	1	5	9	9	$2 = f(1)$
1	1	6	15	24	$= f'(1)$

(6) $f(x) = x^4 + 4x^3 + 10$ $\quad a = -2$

$f'(x) = 4x^3 + 12x^2$

	1	4	0	0	10
−2	1	+2	−4	+8	$-6 = f(-2)$
−2	1	0	−4	16	$= f'(-2)$

(7)$_1$ $f(-2) = 3 \Rightarrow (-2, 3)$

$f'(-2) = -2 = m$

$Y - 3 = -2(x+2)$

$Y - 3 = -2x - 4$

$Y = -2x - 1$

(8)$_2$ $f(4) = -2 \Rightarrow (4, -2)$

$f'(4) = -5 = m$

$Y + 2 = -5(x - 4)$

$Y + 2 = -5x + 20$

$Y = -5x + 18$

(9) $f(2) = 0 \Rightarrow (2,0)$ $y = 15x - 30$
$f'(2) = 15 = m$

(10) $f(3) = 10 \Rightarrow (3,10)$ $y - 10 = 15(x-3)$
$f'(3) = 15 = m$ $y - 10 = 15x - 45$
$y = 15x - 35$

(11) $f(1) = 2 \Rightarrow (1,2)$ $y - 2 = 24x - 24$
$f'(1) = 24 = m$ $y = 24x - 22$

(12) $f(-2) = -6 \Rightarrow (-2,-6)$ $y + 6 = 16(x+2)$
$f'(-2) = 16 = m$ $y + 6 = 16x + 32$
$y = 16x + 26$

(13) $2x + 2 \overset{\text{set}}{=} 0$
$x = -1$
$f(-1) = 1 - 2 + 3 = 2$
tangent horizontal at $(-1, 2)$

(14) $3 - 2x \overset{\text{set}}{=} 0$
$2x = 3$
$x = \frac{3}{2}$

	-1	3	2
$\frac{3}{2}$	-1	$\frac{3}{2}$	$\frac{17}{4}$

tangent horizontal at $\left(\frac{3}{2}, \frac{17}{4}\right)$

(15) $3x^2 + 6x - 9 \overset{\text{set}}{=} 0$
$x^2 + 2x - 3 = 0$
$(x+3)(x-1)$
$x = -3 \quad x = 1$

	1	3	-9	-2
1	1	4	-5	-7
-3	1	0	-9	25

$\{(1,-7), (-3,25)\}$

(16) 4 $3x^2 - 6x + 6 \overset{\text{Set}}{=} 0$

$x^2 - 2x + 2 = 0$

$\frac{2 \pm \sqrt{4-8}}{2}$

Imaginary $\therefore$ No points

(17) 5 $4x^3 + 12x^2 + 8x \overset{\text{Set}}{=} 0$

$x^3 + 3x^2 + 2x = 0$

$x = 0 \qquad x^2 + 3x + 2 = 0$

$(x+1)(x+2)$

$x = -1 \quad x = -2$

	1	4	4	0	-7
0					-7
-1	1	3	1	-1	-6
-2	1	2	0	0	-7

$\{(0, -7), (-1, -6), (-2, -7)\}$

(18) $4x^3 + 12x^2 = 0$ (2nd)

$4x^2(x+3) = 0$

$x = -3$

	1	4	10	0	10
0					10
−3	1	1	−3	9	−17

$\{(0, 10), (-3, -17)\}$

(23) f' exists $(a, f(a))$ $m = f'(a)$

$y - f(a) = f'(a)(x - a)$

$y = xf'(a) + f(a) - af'(a)$

(24) $f(x) = x^2 - 3x + 5$ $(0; 1)$

$f'(x) = 2x - 3$ etc.

Locate critical points. Set $f' \stackrel{set}{=} 0$ & solve.
These are the critical pts.

Rising & Falls. graph critical pts on number graph.
check pts on each side of critical pts. for
the sign of $f'(x)$. + rises, − falls.

find f(critical pts) & pts on each side to determine
relative max, min & pts of inflection
If a, b, c are critical pts.
if $f(a-\varepsilon) < f(a) < f(a+\varepsilon)$ then
a is a pt of inflection it a change
in shape as

$f''(x) = 6x$

$6x > 0$.

$< > 0$

$\cup$

$\curvearrowright$

HW.

① $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$
$f'(x) = 4x^3 - 12x^2 + 12x - 4$
$f''(x) = 12x^3 - 24x + 12$
$f'''(x) = 36x^2 - 24$
$f^{iv}(x) = 72x$
$f^{v}(x) = 72$

② $f(x) = x^5$
$f'(x) = 5x^4$
$f''(x) = 20x^3$
$f'''(x) = 60x^3$
$f^{iv}(x) = 180x^2$
$f^{v}(x) = 360x$
$f^{vi}(x) = 360$

goes back to Pg 136 material.

Lecture:

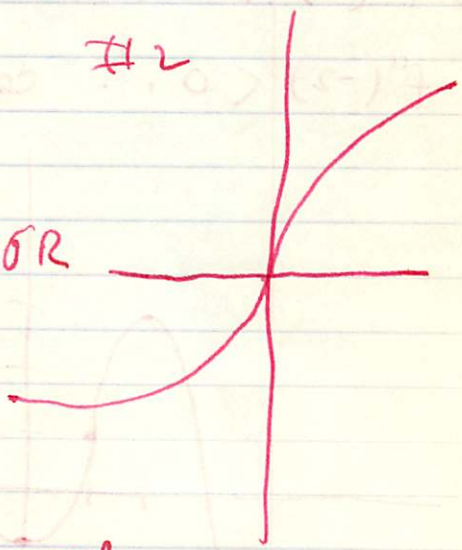
$f(x) = x^3$

which curve is correct?

#1

OR

#2



$f'(x) = 3x^2$

$f''(x) = 6x$

$f''(x) > 0$ when $x > 0$ $\therefore$ $f(x)$ is convex down when $x > 0$

$f''(x) < 0$ when $x < 0$ $\therefore$ $f(x)$ is convex up when $x < 0$.

$\therefore$ #1 is correct Picture.

(Lecture continues over)

$-1 \quad -8$
$+3 \quad 12$
$-1 \quad 4$

$f(x) = x^3 + 3x^2 - 1$

① $f'(x) = 3x^2 + 6x = 3x(x+2)$

② $f''(x) = 6x + 6 = 6(x+1)$

③ $f'(x) = 3x(x+2) \overset{\text{Set}}{=} 0 \quad x \in \{0, -2\}$

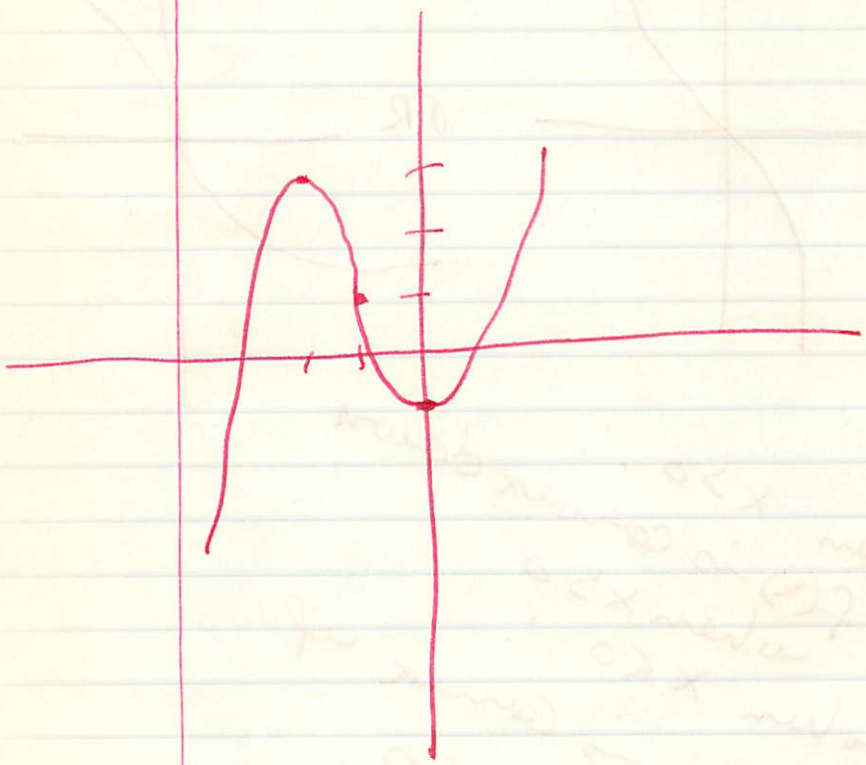
④ $f(0) = -1$, $f(-2) = 3$ } max or min probably. (actually by Second $f''(0) \neq 0$ and $f''(-2) \neq 0$ we do have max or min.

⑤ $f''(x) = 6(x+1) \overset{\text{Set}}{=} 0 \quad x = -1$

⑥ $f(-1) = 1$ $(-1, 1)$ coord of inflection pt.

⑦ $f''(0) > 0$ ∴ convex down & min

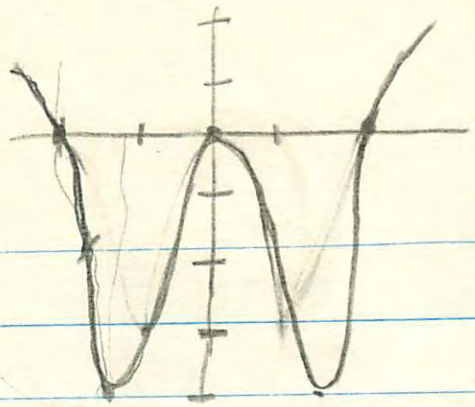
$f''(-2) < 0$ ∴ convex up. & max.



Lecture

$x^2 = \frac{2}{3}$

$x = \pm \frac{\sqrt{6}}{3}$



⑧ $f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

$f''(x) = 12x^2 - 8$

$f'(x) > 0 \quad 4x^3 - 8x > 0$

$x^3 - 2x > 0$

$x^2 - 2 > 0$

$x^2 > 2$

$x > \pm\sqrt{2}$

Critical pts $\{-\sqrt{2}, 0, \sqrt{2}\}$

	1	0	−4	0	0	
−2	1	−2	0	0	0	
* $-\sqrt{2}$	1	$-\sqrt{2}$	−2	$2\sqrt{2}$	−4	
−1	1	−1	−3	3	−3	
* 0	1	0	−4	0	0	
1	1	1	−3	−3	−3	
* $\sqrt{2}$	1	$\sqrt{2}$	−2	$-2\sqrt{2}$	−4	
2	1	2	0	0	0	

less → | greater → | less → | greater →

$-2 \quad -\sqrt{2} \quad -1 \quad 0 \quad 1 \quad \sqrt{2} \quad 2$

$f'(x)$	4	0	−8	0
−2	4	−8	8	−16
~~−3~~	~~4~~	~~−12~~	~~26~~	~~−78~~
−1	4	−4	−4	4
1	4	0	−4	−4
2	4	8	8	16
~~3~~	~~4~~	~~12~~	~~26~~	~~78~~

$f'(x)$ increasing $-\sqrt{2} < x < 0$, $x > \sqrt{2}$

$f'(x)$ Decreasing $-\sqrt{2} < x$, $0 < x < \sqrt{2}$

min at $-\sqrt{2}$, $\sqrt{2}$

max 0

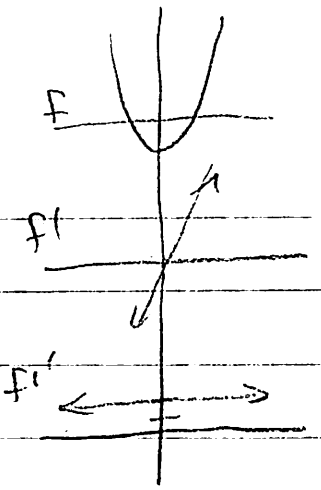
min $\{(-\sqrt{2}, -4), (\sqrt{2}, -4)\}$

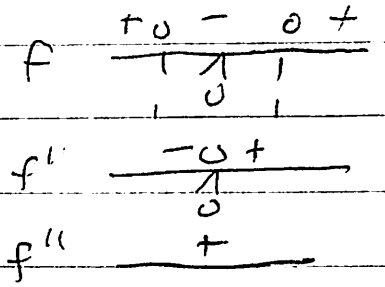
max at $\{0, 0\}$

① $f(x) = x^2 - 1$

$f'(x) = 2x$

$f''(x) = 2$

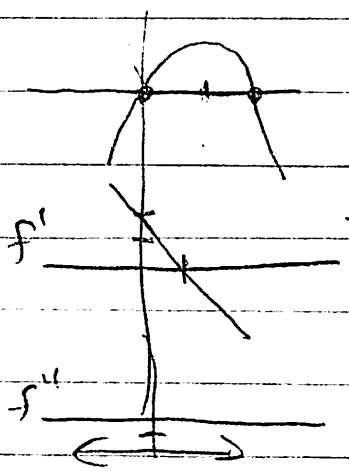


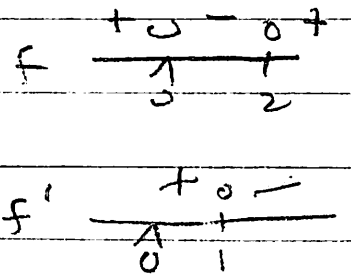


② $f(x) = 2x - x^2 = x(2-x)$

$f'(x) = 2 - 2x = 2(1-x)$

$f''(x) = -2$

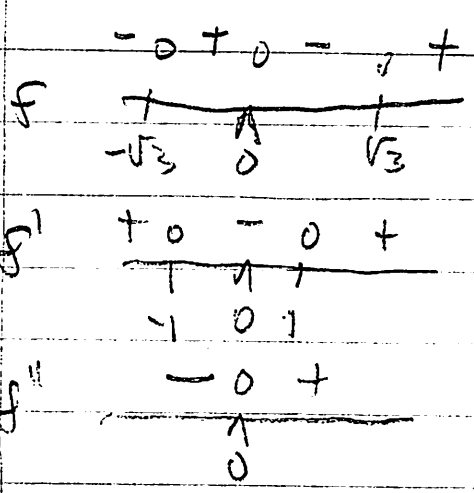


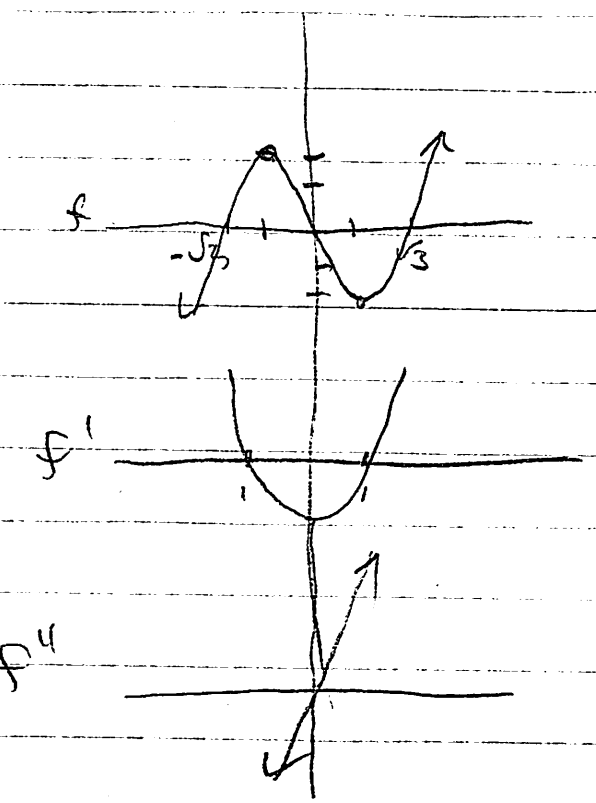


③ $f(x) = x^3 - 3x \quad x(x^2-3)$

$f'(x) = 3x^2 - 3 \quad 3(x-1)(x+1)$

$f''(x) = 6x$





④ $f(x) = x^3 + 3x^2 - 1$

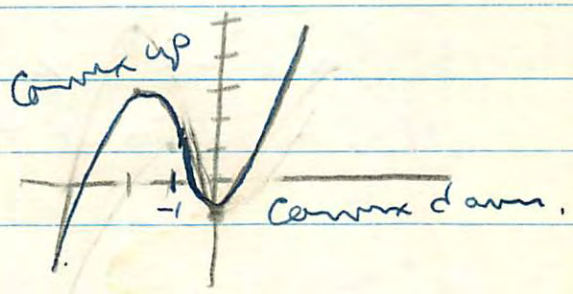
$f'(x) = 3x^2 + 6x$ $\quad 3x^2 + 6x \overset{\text{set}}{=} 0$

$x(x+2) = 0$

$0, -2$

	1	3	0	-1
0				-1 min.
-2	1	1	-2	3 max
-3	1	0	0	-1
-4	1	2	-2	1
1	1	4	0	3

$f''(x) = 6x + 6$



$f'(x) > 0 \quad -2 > x > 0$ Rises

$f'(x) = 0 \quad 0, -2$

$f'(x) < 0 \quad -2 < x < 0$ Falls

⑤ $f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

$f'(x) > 0$ TRs increasing always.

$f'(x) = 0$ NO

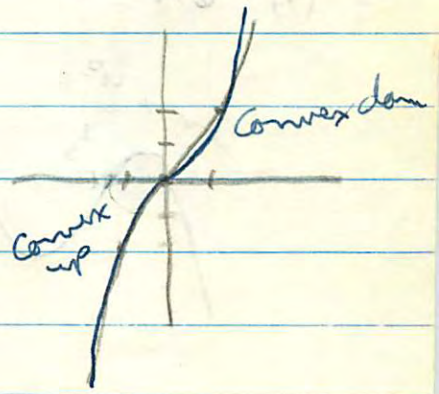
$f'(x) < 0$ NO

$x^2 + 1 = 0$

$x = \pm i$

	1	0	1	0
1	1	1	2	2
-1	1	-1	2	-2
0				0

$f''(x)$ $6x$



No max or min.

(6) $f(x) = x^3 + 1$

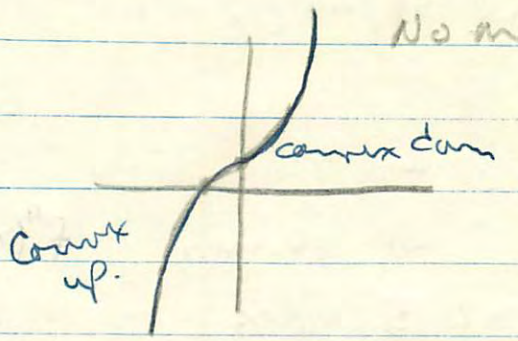
$f'(x) = 3x^2$

always increasing

$f''(x) = 6x$

	1	0	0	1
-1	1	-1	1	0
	0			1
-2	1	-2	4	-7

} change of sign ⇒ No max or min.



(7) $f(x) = x^3 - x^2 - x$

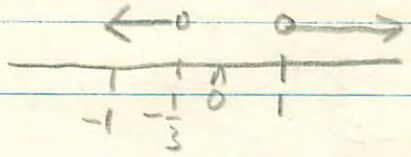
$f'(x) = 3x^2 - 2x - 1$

$3x^2 - 2x - 1 > 0$

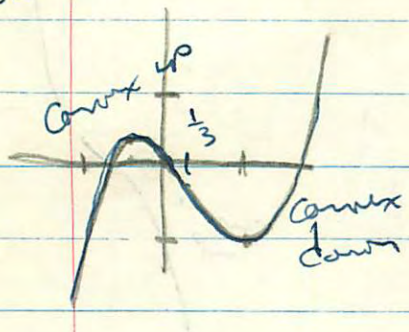
$(3x + 1)(x - 1)$

$x = -\frac{1}{3}$ $x = 1$

	1	-1	-1	0
$-\frac{1}{3}$	1	$-\frac{4}{3}$	$-\frac{5}{9}$	$\frac{5}{27}$
	1	1	0	-1
	0			0
-1	1	-2	1	-1



$f''(x) = 6x - 2$



$f(x) > 0$

	3	-2	-1
1	3	+1	0
2	3	4	7+
-1	3	-5	4+
-2	3	-8	15

Critical pts max $\left(-\frac{1}{3}, \frac{5}{27}\right)$, $(1, -1)$

(1) $f(x) = x^2 - 1$

$f'(x) = 2x$

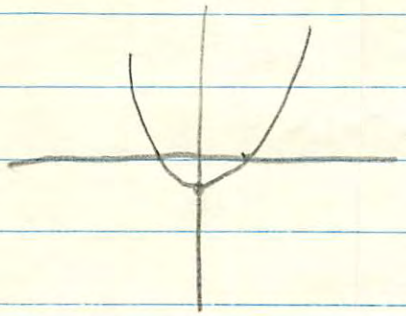
$2x \overset{\text{Set}}{=} 0 \quad x = 0$ ∴ critical pt is $(0, -1)$

$f'(x) > 0$ if $x > 0$ increasing

$f'(0) = 0$ if $x = 0$ min.

$f'(x) < 0$ if $x < 0$ decreasing

	1	0	−1
0	1	0	−1
−1	1	−1	0
	1	0	0



(2) $f(x) = 2x - x^2$

$f'(x) = 2 - 2x = 2(-x + 1) \overset{\text{Set}}{=} 0$

$x = 1 \Rightarrow (2 - 1) = 1$

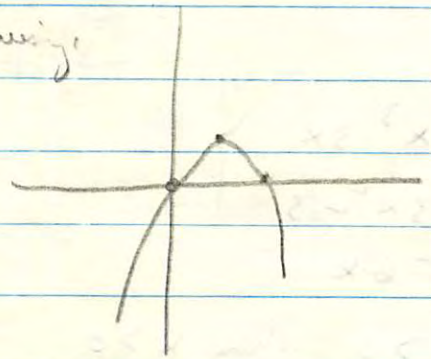
Critical pt $(1, 1)$

$f'(x) > 0$ if $x < 1$ increasing

$f'(x) = 0$ if $x = 1$ critical

$f'(x) < 0$ if $x > 1$ decreasing

	−1	2	0
1	−1	+1	1
0			0
2	−1	0	0



(3) $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3$

$x^2 - 1 = 0$

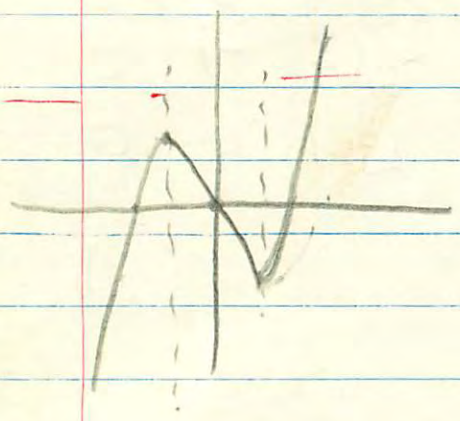
$x = \pm 1$

	1	0	-3	0
1	1	0	-2	-2
-1	1	-1	-2	2
	0			0

$f'(x) > 0 \quad |x| > 1$ increasing

$f'(x) = 0 \quad x = 1$

$f'(x) < 0 \quad |x| < 1$ decreasing



$x^2 - 1 > 0$

$x^2 - 1 < 0$

$x^3 - 3x = 0$

$x^2 - 3 = 0$

$(x - \sqrt{3})(x + \sqrt{3})$

rel max at $(-1, 2)$

rel min at $(1, -2)$

	1	0	-3	0
2	1	2	1	2
-2	1	-2	1	-2

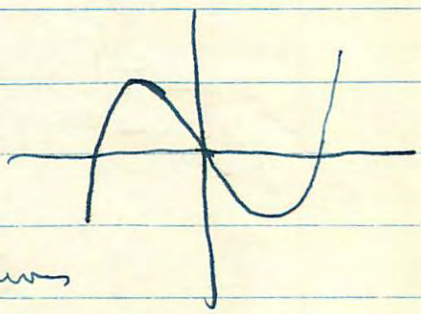
Pg 142 #3 $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

$f''(x) > 0$ when $x > 0$ Convex down

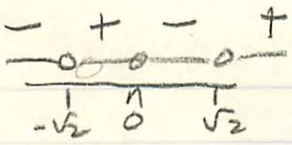
$f''(x) < 0$ when $x < 0$ Convex up



$f''(x) = 12x^2 - 8$ $\quad x = \pm\sqrt{\frac{2}{3}}$

$4(3x^2 - 2)$

(8) $f(x) = x^4 - 4x^2$

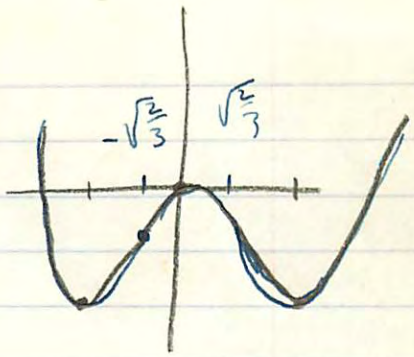


$f'(x) = 4x^3 - 8x$

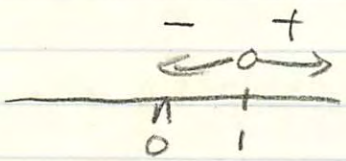
$4x(x^2 - 2)$

$0, \pm\sqrt{2}$

	1	0	−4	0	0
$-\sqrt{2}$	1	$-\sqrt{2}$	−2	$2\sqrt{2}$	−4
$+\sqrt{2}$	1	$\sqrt{2}$	−2	$-2\sqrt{2}$	−4



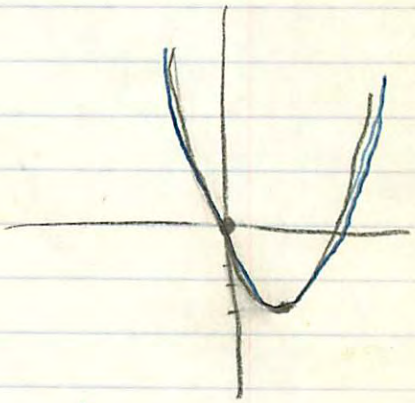
(9) $f(x) = x^4 - 4x$ $\quad x(x^3 - 4)$



$f'(x) = 4x^3 - 4$

$4(x^3 - 1)$

$f(1) = -3$



$f''(x) = 12x^2 > 0 \ \forall \ x \in \mathbb{R}$ $\quad \therefore$ Concave down

10 $f(x) = x^4 + 4x^3 + 7$

$f'(x) = 4x^3 + 12x^2$

$4x^3 + 12x^2 \overset{\text{Set}}{=} 0$

$x + 3 = 0$

$x = -3 \quad 0$

	1	4	0	0	7	
-4	1	0	0	0	7	
* -3	1	1	-3	9	-20	
-2	1	-1	-1	5	-12	
-1	1	3	-3	3	4	
* 0					7	
	1	1	5	5	5	12

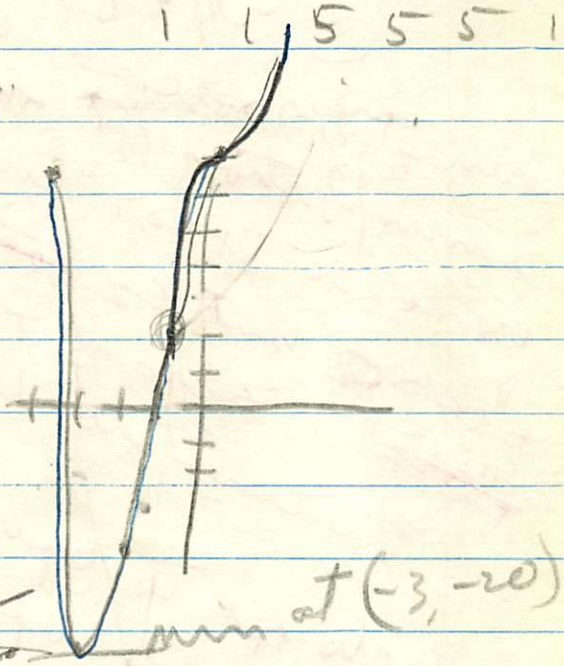
$f''(x) = 12x^2 + 12x$

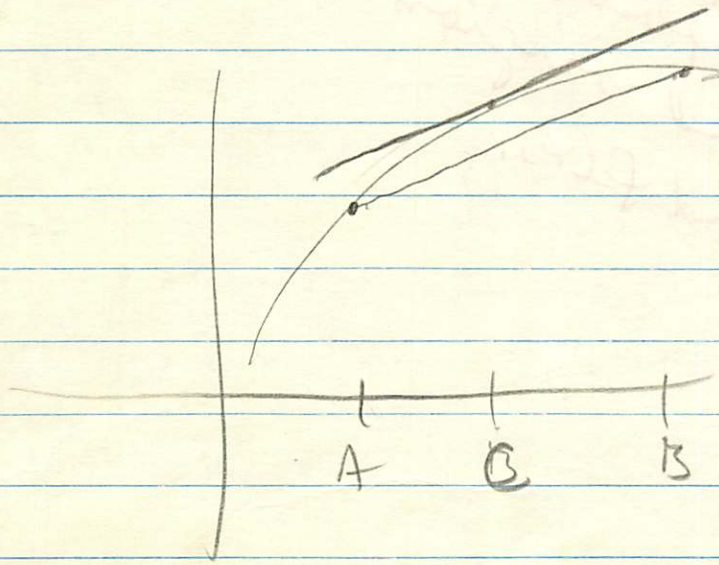
$12x(x+1)$

$x \leq -1 \quad f''(x) \geq 0$ convex down

$0 < x < -1 \quad f(x) < 0$ convex up

$x > 0 \quad f(x) > 0$ convex down





$\frac{12}{2}$

$\frac{9}{2}$

$3\left(\frac{3}{4}\right)^2$ $-\frac{3}{2}\cdot\frac{3}{4} = -\frac{9}{8}$

$-\frac{12}{2}$

$3\left(\frac{1}{4}\right)^2 = \frac{3}{2}$

$-\frac{9}{2} = -\frac{9}{8}$

$\frac{1}{4}$

$6\cdot\frac{3}{4}$

$\frac{9}{2}$ $4\frac{1}{2}$

$\frac{3}{2}$

$\frac{3}{4}$ 12 9 48

$\frac{3}{4}$ $\frac{27}{4}$

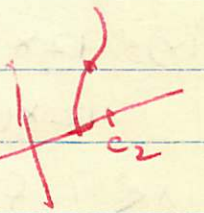
$\frac{21}{4} = -\frac{63}{16}$

$\frac{3}{4}$

CHANGE

when $f'(x) = 0 \Rightarrow c_1 = 0$ but c_1 is an inflection pt in f we have a "LAZY" curve such as →

but c_2 does not a $f''(x) = 0$ but zero of f', then



the curve is "very quick" such as

Make change to equations that has Both types of inflection pts.

also try to avoid singularities in Both $f'(x)$ and $f(x)$.

almost last lecture Chapt 3 ~~1st lecture Chapt 4 etc.~~

$f(x) = x^4 - 6x^2 - 8x + 7 \quad [-2, 3]$

$f'(x) = 4x^3 - 12x - 8$

$f'(x) = (x+1)^2(x-2)$

critical values of x: $c \in \{-1, 2\}$

	4	0	-12	-8
-1	4	-4	-8	0
-1	4	-8	0	
2	4	0		

0 0

-1 0 2

* three choices are:
① convenient
② Do not "skip over" a critical value.

f test

	1	0	-6	-8	7
* -2	1	-2	-2	-4	15
-1	1	-1	-5	-3	10
0					7
2	1	2	-2	-12	-17
* 3	1	3	+3	+1	10

$15 < 10 < 7$ ∴ PI at $(-1, 10)$

$-17 < 10$ & $-17 < 10$ ∴ min at $(2, -17)$

f' test.

$f' = (x+1)^2(x-2) \Rightarrow$

I II III

− ⊗ − 0 +

-1 0 2

Since interval I, II have same sign, $(-1, 10)$ is PI

Since interval II, III goes from − to + is − to 0 to +

f'' test.

$f'' = 12x^2 - 12$

$f''(x) = 12(x^2 - 1) \Rightarrow$

0 0

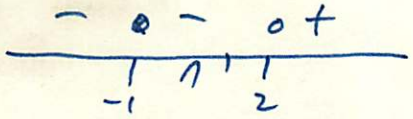
-1 +1

	12	0	-12	
-1	12	-12	0	← no conclusion is possible.
2	12	24	36	← >0 ∴ min at $(2, -17)$

If $f''(c) > 0$ min at $(c, f(c))$; if $f''(c) < 0$, MAX at $(c, f(c))$

if $f''(c) = 0$, TEST fails.

Since this is a "bounded" fctn in $[-2, 3]$.
we must consider the end pts as possible Extremum.

Use $f' = (x+1)^2(x-2) \Rightarrow$ 

Since $f' < 0$ "JUST "After" $x = -2$ we say "it decreases from $(-2, f(-2))$ $\therefore$ $f(-2)$ is a maximum

Since $f' > 0$ "Just before" $x = 3$ we say "it increases to $(3, f(3))$

final conclusions.

$(-2, 15)$ Abs max

$(-1, 10)$ PI

$(2, -17)$ abs min

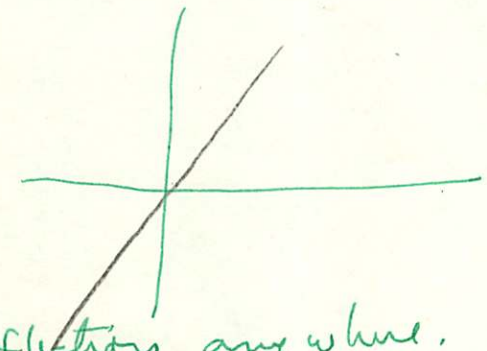
$(3, 10)$ rel max (only because $10 < 15$.)

FINAL Note:

Consider $f(x) = x$

$f'(x) = 1$

$f''(x) = 0$



obviously, not change of inflection anywhere.

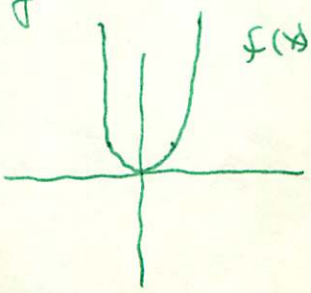
---

consider $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f'(x) \overset{\text{Let}}{=} 0 \Rightarrow x = 0$ $f''(0) = 0$



El Quizzo

Name _____

No Calculators

Per _____ Date _____

I Given $f(x)=3x^4-8x^3+6x^2-1$ for $-1\le x\le 2$.

Determine and identify all significant points (ie. extrema, PI, intercepts, etc)

	3	−8	6	0	−1
−1	3	−11	17	−17	16
					−1
0					
$\frac{1}{3}$	3	−7	$\frac{11}{3}$	$\frac{11}{9}$	$-\frac{16}{27}$
1	3	−5	1	1	0
2	3	−2	2	4	7

$f' = 12x^3-24x^2+12x$

$12x(x^2-2x+1)$

$12x(x-1)^2$

$f'' = 36x^2-48x+12$

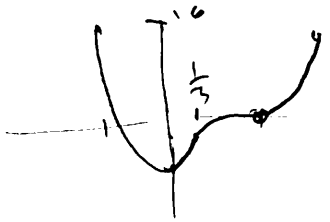
$12(3x^2-4x+1)$

$12(3x-1)(x-1)$

f': − 0 + 0 +

f'': + 0 − 0 +

0 $\frac{1}{3}$ 1



$(-1, 16)$ max, END PT of f, def int

$(0, -1)$ min, f int

$(\frac{1}{3}, -\frac{16}{27})$ PI, change fm cu up to cu dn

$(1, 0)$ LPI, change fm cu dn to cu up.

$(2, 7)$ max, END PT of f

II Find the limit:

a) $\lim_{x\to 0}\left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

$\lim_{x\to 0}\left(\frac{3+2x}{x(1+5x)}-\frac{3(1+5x)}{x(1+5x)}\right)$

$\lim_{x\to 0}\frac{-13x}{x(1+5x)} = -13$

b) $\lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{x}\cdot\frac{(\sqrt[3]{\ })^2+\sqrt[3]{\ }+1}{(\ )+\sqrt{\ }+1}$

$\lim_{x\to 0}\frac{x}{x\left((\sqrt[3]{x+1})^2+\sqrt[3]{x+1}+1\right)}$

$\frac{1}{3}$

Grade Had them offer 4 checks if all problems are explained in narrative fashion — Due Day of Exam

Turn over for part III

III State and identify both "definitions of the derivative."

Explain how they are different.
Explain how they are the same.
Explain when each is used. Be complete.
Demonstrate each definiton of the derivative

using the function $f(x)=\frac{x^2+1}{x}$ and the point $\left(2,\frac{5}{2}\right)$

1st $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$

2nd $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$

1st yields number used to plug in DUE
2nd yields ~ f of x
Both have results of the slope formula $\frac{y_2-y_1}{x_2-x_1}$

$$\lim_{x\to\frac{5}{2}}\frac{\frac{x^2+1}{x}-\frac{5}{2}}{x-2}$$

$$\lim_{x\to\frac{5}{2}}\frac{2x^2+2-5x}{2x(x-2)}$$

$$\lim_{x\to\frac{5}{2}}\frac{(2x-1)(x-2)}{2x(x-2)}$$

$$\frac{3}{4}$$

$$\lim_{h\to 0}\frac{1}{h}\left(\frac{(x+h)^2+1}{x+h}-\frac{x^2+1}{x}\right)$$

$$\lim_{h\to 0}\frac{1}{h}\left[\frac{x(x^2+2hx+h^2+1)-(x^2+1)(x+h)}{x(x+h)}\right]$$

$$\lim_{h\to 0}\frac{x^3+2hx^2+h^2x+x-(x^3+hx^2+x+h)}{hx(x+h)}$$

$$\lim_{h\to 0}\frac{x^3+2hx^2+h^2x+x-x^3-hx^2-x-h}{hx(x+h)}$$

$$\lim_{h\to 0}\frac{h(x^2+hx-1)}{hx(x+h)}$$

$$\frac{x^2-1}{x^2}\Big|=\frac{3}{4}$$

$x=2$

Making quiz

AP Calc ch 3

$f(x) = 3x^4 - 8x^3 + 6x^2 - 1$

$f'(x) = 12x(x-1)^2$

$f''(x) = 12(3x-1)(x-1)$

	−	0	+	0	+
f'					
	0		$\frac{1}{3}$		1

	+	0	−	0	+
f''					
	0		$\frac{1}{3}$		1

min at (0, 1)

LAZY PI (1, 2)

Actual PI $\frac{1}{3}, \frac{38}{27}$

	3	−8	6	0	−1
−1	3	−11	17	−17	16
1	3	−5	1	1	0
$-\frac{1}{3}$	3	−9	9	−3	0
$-\frac{1}{3}$	3	−6	3	0	
	1	−2	1		

$(x-1)^2$

$(x-1)^3(3x+1)$

I
II

Given $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$ for $-1 \le x \le 2$

Determine and identify all significant pts. ie extrema, PI, intercepts.

	3	−8	6	0	−1
−2	3	−14	34	−68	
−1	3	−11	17	−17	16
2	3	−2	2	4	7

Offer student who missed/ng Quiz 4 MINUSES instead of this Quiz.
Normal grading on this = 4 =.

f(x) =

Find the following limits.

III $\lim_{x\to 0}\left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

$\lim_{x\to 0}\frac{3+2x}{x(1+5x)}-\frac{3}{x}$

$\lim_{x\to 0}\frac{3+2x-3-15x}{x(1+5x)}$

$\lim_{x\to 0}\frac{-13x}{x(1+5x)} = -13$

$\lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{x}\left(\frac{(\quad)^2+\sqrt[3]{\quad}+1}{(\quad)}\right)$

$\lim_{x\to 0}\frac{x+1-1}{x\left((\sqrt[3]{x+1})^2+\sqrt[3]{x+1}+1\right)}$

$\frac{1}{3}$

IV ~~Find f' and f'' using~~ State 1st and 2nd Def of Der. Explain how they are different and when they are used. Explain how they are the same. Be complete.

Demonstrate each ~~given~~ on the fcn $f(x) = \frac{x^2+1}{x}$ and the point $(2, \frac{5}{2})$